1) a) $|x+2| > \frac{1}{3}|x-2|$

	α	-2	β	2	γ
$\lvert x+2\rvert$	$-x-2$		$x+2$		$x+2$
$\lvert x-2\rvert$	$-x+2$		$-x+2$		$x-2$

$|x+2| = 0 \qquad x = -2$

$|x-2| = 0 \qquad x = 2$

α) $x < -2$

$|x+2| = -(x+2)$

$|x-2| = -(x-2)$

$$-x-2 > \frac{1}{3}(-x+2)$$

$$-x-2 > -\frac{1}{3}x + \frac{2}{3} \quad \Big| + \frac{1}{3}x \quad \Big| +2$$

$$-\frac{2}{3}x > 2\frac{2}{3} \quad \Big| : \left(-\frac{2}{3}\right)$$

$$\underline{\underline{x < -4}} \qquad x \in (-\infty, -4)$$

β) $-2 \leq x \leq 2$

$|x+2| = (x+2)$

$|x-2| = -(x-2)$

$$x+2 > \frac{1}{3}(-x+2)$$

$$x+2 > -\frac{1}{3}x + \frac{2}{3} \quad \Big| + \frac{1}{3}x \quad \Big| -2$$

$$1\frac{1}{3}x > -1\frac{1}{3} \quad \Big| : 1\frac{1}{3}$$

$$x > -1$$

$x \in (-1, \infty)$, i.e $x \in (-1, 2]$

Bereich beachten

γ) $x > 2$

$|x+2| = (x+2)$

$|x-2| = (x-2)$

$$x+2 > \frac{1}{3}(x-2)$$

$$x+2 > \frac{1}{3}x - \frac{2}{3} \quad \Big| -\frac{1}{3}x \quad \Big| -2$$

$$\frac{2}{3}x > -\frac{8}{3}, \quad x > -4; \text{ i.e. } x > 2$$

(Bereich beachten)

$$\underline{\underline{x \in (-\infty, -4) \cup (-1, \infty)}} \quad \checkmark$$

1 b) $\dfrac{3x+2}{2x-1} < 2 \quad |\cdot(2x-1)$

α) β)
$-$ $\frac{1}{2}$ $+$ $\quad 2x-1$

β) $x > 1/2$

$3x+2 < 2(2x-1)$

$3x+2 < 4x-2 \quad |+2$

$3x+4 < 4x \quad |-3x$

$4 < x$

$\underline{\underline{x > 4}}$

α) $x < 1/2$

d.h. $2x-1 < 0 \Rightarrow$

$3x+2 > 4x-2$

$\underline{\underline{4 > x}}$,

d.h. $\forall x < 1/2$ (Bereich beachten)

Gesamtlösung: $x \in (-\infty, 1/2) \cup (4,\infty) = \mathbb{R}^1 \setminus [1/2, 4]$.

2 a) $z = \dfrac{(1+2i)(i-1)+1}{(3+2i)^2 - 2(2+i)}$

NR: $(1+2i)(i-1) = i - 1 + 2i^2 - 2i = -i - 3$

$(3+2i)(3+2i) = 9 + 12i + 4i^2 = 12i + 5$

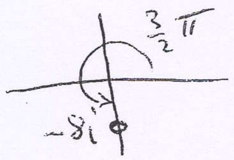
$z = \dfrac{-i-3+1}{12i+5-4-2i} = \dfrac{-i-2\,(1-10i)}{(10i+1)(1-10i)}$

$= \dfrac{-i+10i^2-2+20i}{-100i^2+1} = \underline{\underline{\dfrac{-12+19i}{101}}}$ ✓

$Re_{(z)} = \dfrac{-12}{101} \qquad Im(z) = \dfrac{19}{101}$

2b) $z^3 + 8i = 0$

$z^3 = -8i$



$x = r \cdot \cos\varphi \qquad r = 8$

$y = r \cdot \sin\varphi \qquad r = \sqrt{x^2+y^2}$

$0 = 8 \cdot \cos\varphi$

$-8 = 8 \cdot \sin\varphi$

$\sin\varphi = -1 \qquad \varphi = \frac{3}{2}\pi$

$\cos\varphi = 0$

$z_k = \sqrt[3]{8}\left(\cos\left(\dfrac{\frac{3}{2}\pi + 2k\pi}{3}\right) + i\sin\left(\dfrac{\frac{3}{2}\pi + 2k\pi}{3}\right)\right), \; k = 0,1,2$

$z_0 = 2\left(\cos \pi/2 + i \sin \pi/2\right) = 2\cdot 0 + 2i = \underline{\underline{2i}}$

$z_1 = 2\left(\cos \frac{7}{6}\pi + i \sin \frac{2\pi}{6}\right) = -\sqrt{3} - i, \quad z_2 = 2\left(\cos \frac{11\pi}{6} + i \sin \frac{11}{6}\pi\right) =$

$= \sqrt{3} - i$

Leistungskontrolle Mathematik

3) $AX + 2B = A^T - 2X$

$A = \begin{bmatrix} 5 & 2 \\ 4 & -1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$

$A^T = \begin{bmatrix} 5 & 4 \\ 2 & -1 \end{bmatrix}$

$AX + 2X = A^T - 2B$

$(A + 2E)X = A^T - 2B$

$x = (A + 2E)^{-1} \cdot (A^T - 2B)$

$A + 2E = \begin{bmatrix} 5 & 2 \\ 4 & -1 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix}$

$(A + 2E)^{-1}$:

$$\begin{array}{rr|rr} 7 & 2 & 1 & 0 \\ 4 & 1 & 0 & 1 \\ \hline -1 & 0 & 1 & -2 \\ 4 & 1 & 0 & 1 \\ \hline -1 & 0 & 1 & -2 \\ 0 & 1 & 4 & -8 \\ \hline 1 & 0 & -1 & 2 \\ 0 & 1 & 4 & -7 \end{array}$$

$(A + 2E)^{-1} = \begin{bmatrix} -1 & 2 \\ 4 & -7 \end{bmatrix}$

$= \frac{1}{|A+2E|} \begin{bmatrix} 1 & -2 \\ -4 & 7 \end{bmatrix} = -1 \begin{bmatrix} 1 & -2 \\ -4 & 7 \end{bmatrix}$

$A^T - 2B = \begin{bmatrix} 5 & 4 \\ 2 & -1 \end{bmatrix} - 2 \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$

$x = (A + 2E)^{-1} \cdot (A^T - 2B)$

$$\begin{array}{rr|rr} & & 1 & -2 \\ & & 0 & 3 \\ \hline -1 & 2 & -1 & 8 \\ 4 & -7 & 4 & -29 \end{array}$$

$x = \begin{bmatrix} -1 & 8 \\ 4 & -29 \end{bmatrix}$

Kontrolle: $A \cdot x + 2B = A^T - 2 \cdot X$

$A \cdot X$:

$$\begin{array}{rr|rr} & & -1 & 8 \\ & & 4 & -29 \\ \hline 5 & 2 & 3 & -18 \\ 4 & -1 & -8 & 61 \end{array}$$

$AX + 2B = \begin{bmatrix} 3 & -18 \\ -8 & 61 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 7 & -12 \\ -6 & 57 \end{bmatrix}$

$A^T - 2 \cdot x = \begin{bmatrix} 5 & 4 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} -2 & 16 \\ 8 & -58 \end{bmatrix} = \begin{bmatrix} 7 & -12 \\ -6 & 57 \end{bmatrix}$

4)

$$\text{I} \quad 1x + 3y + (a-1)z = 0$$
$$\text{II} \quad 2x + 1y + 1z = 0$$
$$\text{III} \quad 1x + 2y + az = 0$$

	x	y	z	
*	1	3	a−1	0
−2	2	1	1	0
−1	1	2	a	0
−5	0	−5	−2a+3	0
*	0	−1	1	0
	0	0	−2a−2	0

für $a = \cancel{1}\,-1$ existieren unendlich viele Lösungen $R(A) = 2$

1 Variable frei wählbar

$\underline{z = t}$, 2. Sternzeile $-y + t = 0$

$\Rightarrow \underline{y = t}$

1. Sternzeile:

$x + 3t - 2t = 0 \quad x = -t, \begin{pmatrix} x \\ y \\ z \end{pmatrix} = t \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix},$

$t \in \mathbb{R}$

5)

$\bar{F} = [11, 13, 14]^T$

$\bar{a}_1 = [1, 3, 2]^T$, $\bar{a}_2 = [2, 1, 3]^T$ und $\bar{a}_3 = [3, 4, 1]^T$

$$\bar{F} = \lambda_1 \bar{a}_1 + \lambda_2 \bar{a}_2 + \lambda_3 \bar{a}_3$$

	λ_1	λ_2	λ_3	b
*	1	2	3	11
−3	3	1	4	13
−2	2	3	1	14
*	0	−5	−5	−20
−1	0	−1	−5	−8
*	0	4	0	12

3. Sternzeile

$4\lambda_2 = 12 \Rightarrow \underline{\lambda_2 = 3}$

2. Sternzeile

$-5\lambda_2 - 5\lambda_3 = -20$, d.h.

$-15 - 5\lambda_3 = -20$

$-5\lambda_3 = -5, \ \underline{\lambda_3 = 1}$

1. Sternzeile

$\lambda_1 + 2\lambda_2 + 3\lambda_3 = 11$

$\lambda_1 + 6 + 3 = 11$

$\underline{\lambda_1 = 2}$

Lösung

$$\begin{pmatrix} 11 \\ 13 \\ 14 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + 3 \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + 1 \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$$

6.) geg: $P_1(3;4;5)$, $P_2(4;5;8)$, $P_3(5;3;7)$

a) ges: Ebene E durch P_1, P_2, P_3 (Parameterdarstellung)

$$E: \bar{r} = \overline{OP_1} + s\,\overline{P_1P_2} + t\,\overline{P_1P_3} \qquad (s,t \in \mathbb{R})$$

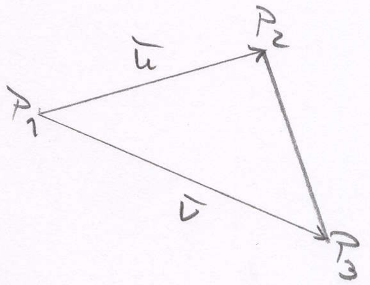
$$= \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$

+, -

b) ges: Flächeninhalt A des Dreiecks $P_1P_2P_3$

$$\bar{u} = \overline{P_1P_2} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$$

$$\bar{v} = \overline{P_1P_3} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$



$$\bar{u} \times \bar{v} = \begin{vmatrix} \bar{e}_1 & \bar{e}_2 & \bar{e}_3 \\ 1 & 1 & 3 \\ 2 & -1 & 2 \end{vmatrix} = \bar{e}_1 \begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} - \bar{e}_2 \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} + \bar{e}_3 \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}$$

$$= 5\bar{e}_1 + 4\bar{e}_2 - 3\bar{e}_3 = \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}$$

$$A = \frac{1}{2}|\bar{u} \times \bar{v}| = \frac{1}{2}\sqrt{25+16+9} = \frac{1}{2}\sqrt{50} = \underline{\frac{5}{2}\sqrt{2} = 3{,}53}$$

7.) geg: $E_1: 3x - 2y + z = 5$

a) ges: Spiegelpunkt P' von $P(-4; 5; -1)$ bzgl. E_1

Sei g = Gerade durch P senkrecht zu E_1

$g: \vec{r} = \overline{OP} + t\vec{n}$ mit $\vec{n} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ - Normalenvektor von E_1

$= \begin{pmatrix} -4 \\ 5 \\ -1 \end{pmatrix} + t \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$

Durchstoßpunkt: g in E_1 einsetzen

$\vec{n} \cdot \vec{r} = 5$

$\vec{n} \cdot \overline{OP} + t_s \vec{n} \cdot \vec{n} = 5$

$t_s = \dfrac{5 - \vec{n} \cdot \overline{OP}}{\vec{n}\,\vec{n}} = \dfrac{5 - (-12 - 10 - 1)}{9 + 4 + 1} = \dfrac{28}{14} = 2$

$\Rightarrow$ Durchstoßpunkt P_s: $\overline{OP_s} = \overline{OP} + t_s \vec{n}$

$= \begin{pmatrix} -4 \\ 5 \\ -1 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

Spiegelpunkt P': $\overline{OP'} = \overline{OP} + 2t_s\vec{n} = \begin{pmatrix} -4 \\ 5 \\ -1 \end{pmatrix} + 4 \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ -3 \\ 3 \end{pmatrix}$

b) ges: $E_2 \parallel E_1$ mit $R(2; 3; 4) \in E_2$

NV $\vec{n}_2$ von E_2: $\vec{n}_2 \parallel \vec{n}$, d.h. $\vec{n}_2 = \lambda \vec{n}$ $(\lambda \in \mathbb{R})$ $\lambda \neq 0$

E_2: $\vec{n}_2 \cdot \vec{r} = \vec{n}_2 \cdot \overline{OR}$

$\lambda(3x - 2y + z) = \lambda(6 - 6 + 4)$ $\quad | : \lambda$

$\underline{3x - 2y + z = 4}$