

Tutorial Stochastic Processes
(Serie 1, 2013/14 Basics)

1. The random variable X is defined by $P(X = -2) = 1/3$,
 $P(X = 1) = 1/2$ and $P(X = 2) = 1/6$. Calculate the distribution
function $F_X(x)$, the expectation $E(X)$ and the variance $\text{Var}(X)$.
2. Let Y be the face value on a fair die. Calculate the expectation $E(Y)$
and the variance $\text{Var}(Y)$.
3. Let U be the continuous uniform r.v. on the interval $(2, 6)$. Calcula-
te the distribution function $F_U(x)$, the expectation $E(U)$ and the
variance $\text{Var}(U)$.
4. Let $X \sim \text{Exp}(\lambda)$. Prove that the exponential distributed r.v. X is
memoryless, i.e. $P(X > x + y | X > x) = P(X > y)$ for all $x > 0$ and
 $y > 0$, where $P(A|B) = \frac{P(A \cap B)}{P(B)}$ denotes the conditional probability.
5. Let X_i be $N(i, i^2)$ -distributed, $i = 1, 2, 3$, and let X_1, X_2, X_3 be inde-
pendent.

Define $Y_1 = X_1 - X_2$, $Y_2 = X_3 - X_1$ and $Y_3 = X_1$.

Find the expectation vectors and covariance-matrices $E(X_1, X_2, X_3)$,
 $\Sigma_{(X_1, X_2, X_3)}$, $E(Y_1, Y_2, Y_3)$ and $\Sigma_{(Y_1, Y_2, Y_3)}$!

References to Stochastic Processes

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