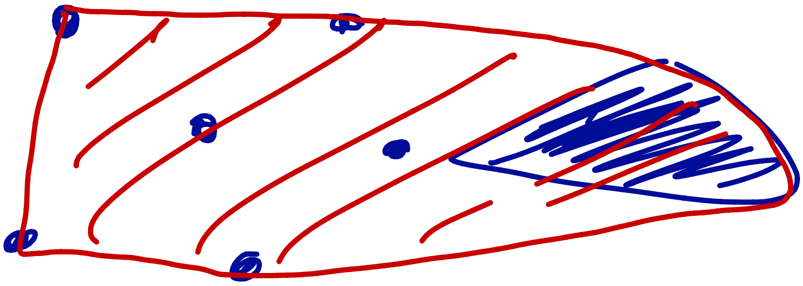
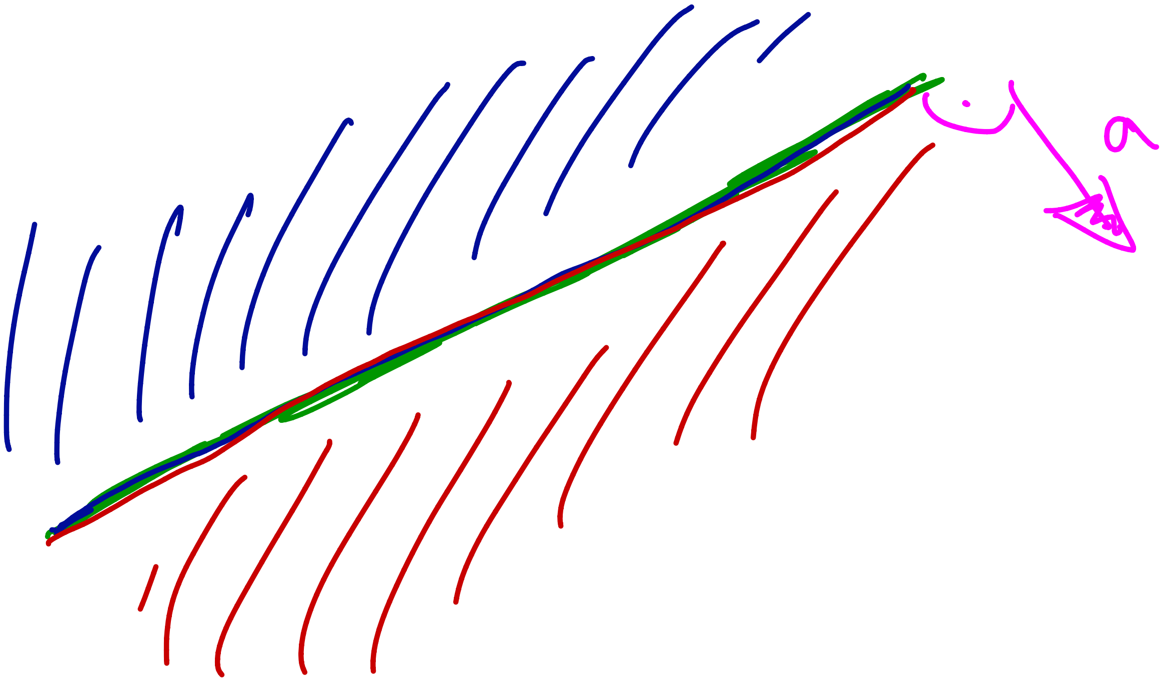
EMO 18.10.16

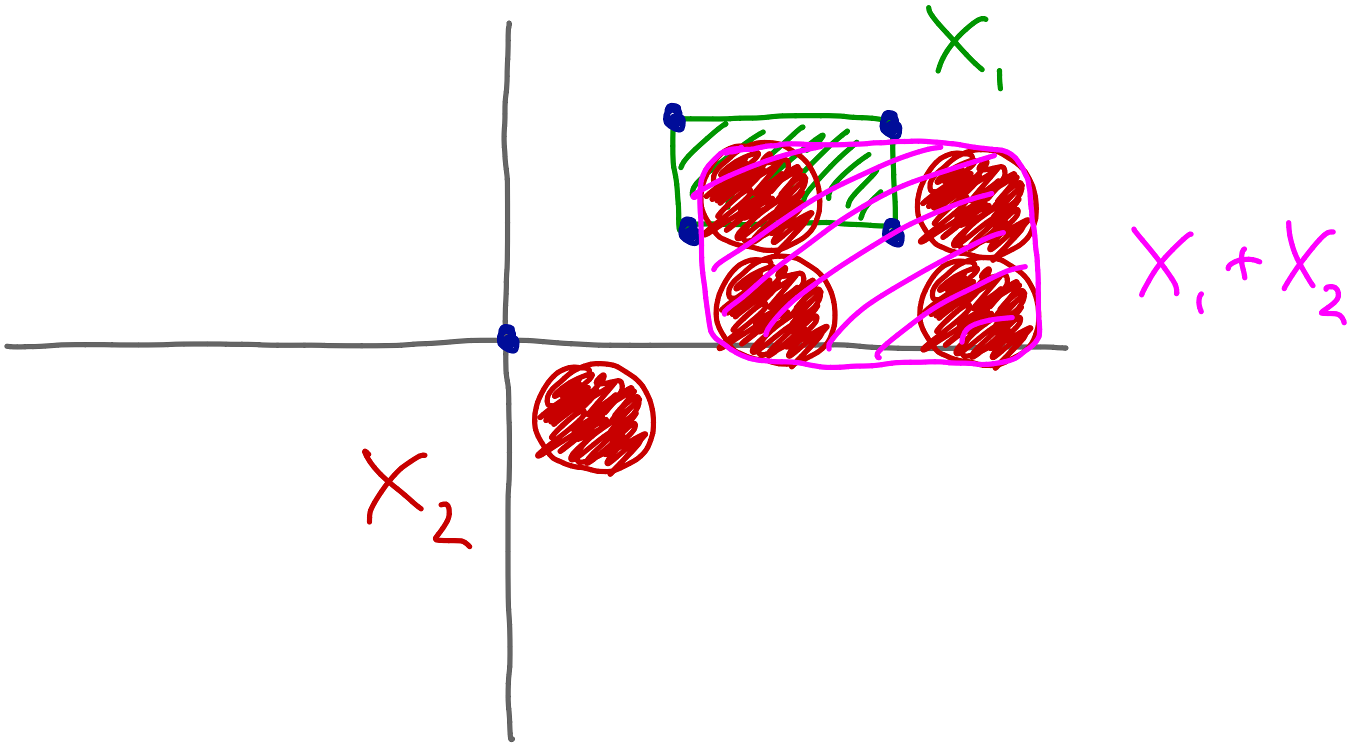
X

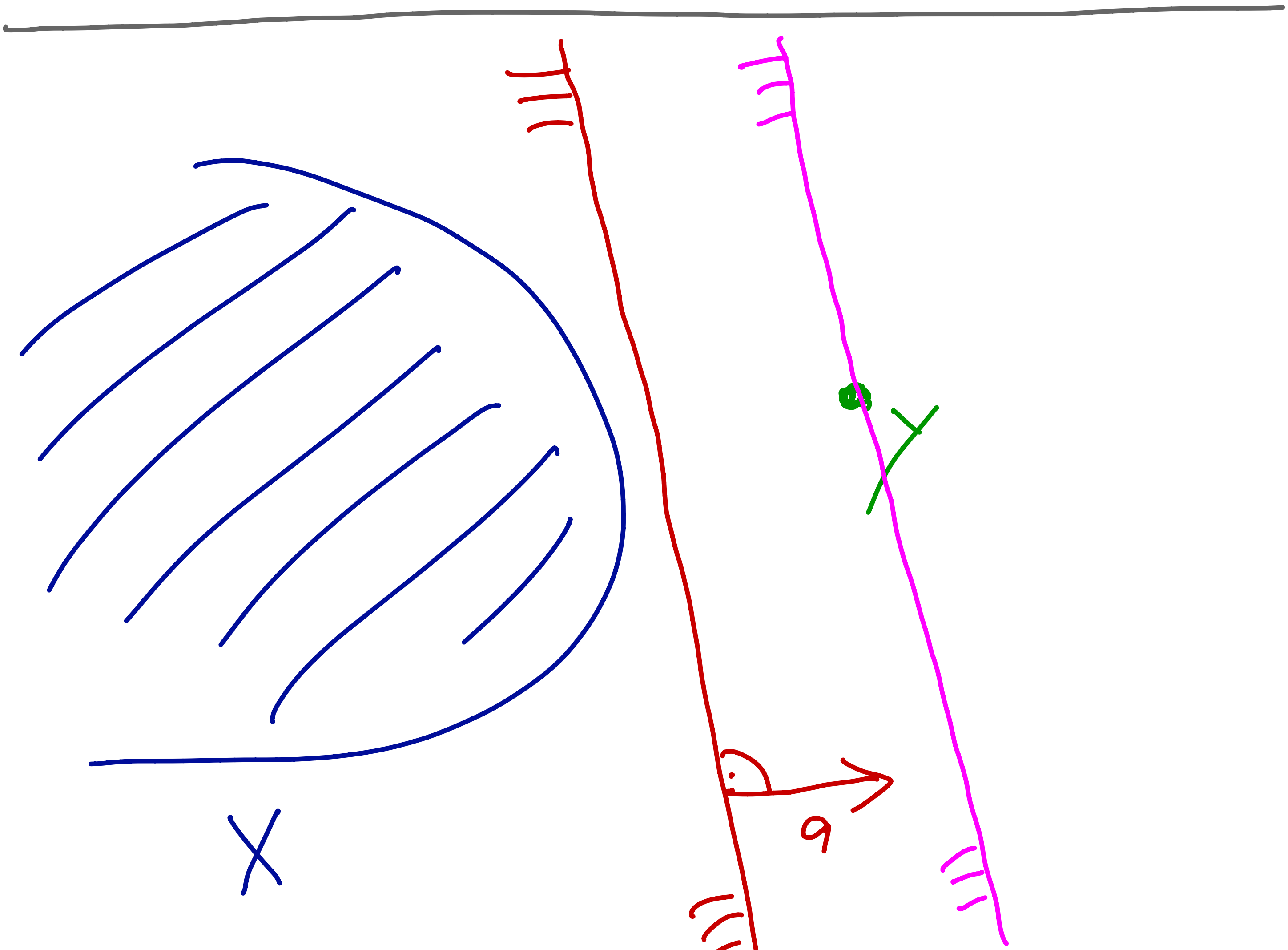


conv X

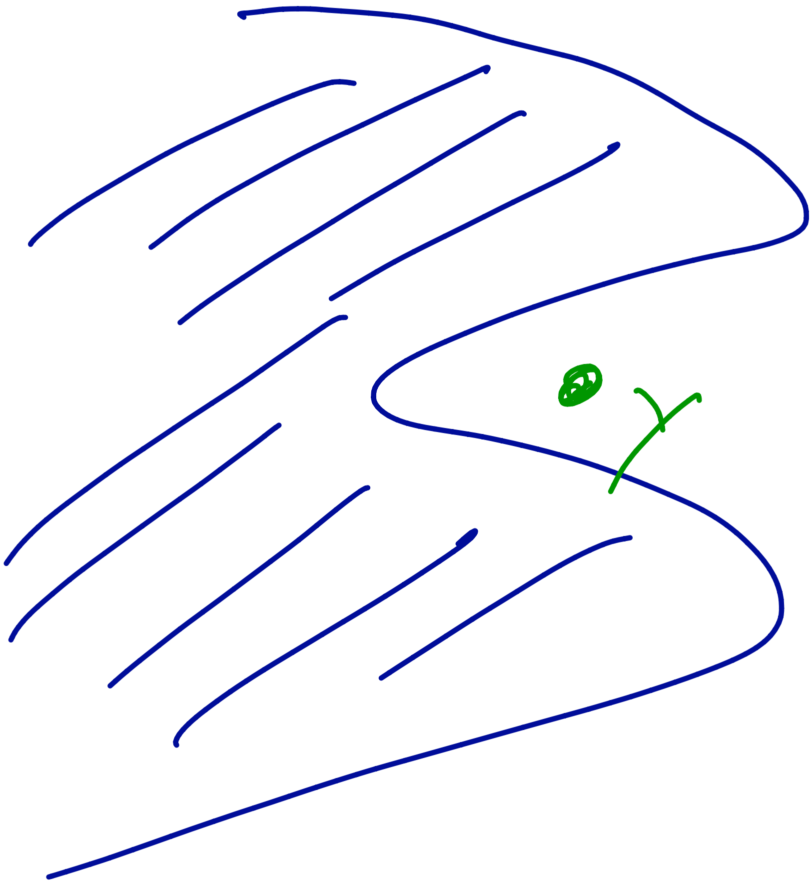
$$\boxed{H^{=}(a,\beta)} = \boxed{H^{\le}(a,\beta)} \cap \boxed{H^{\le}(-a,-\beta)}$$



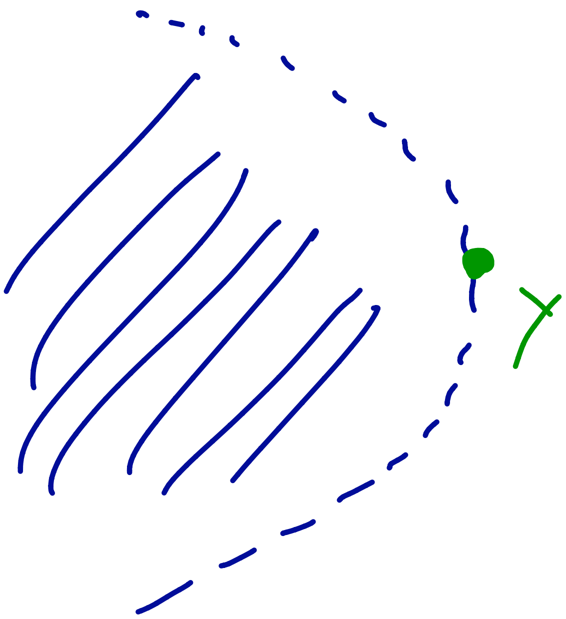




$\{x : \langle a, x \rangle \leq \langle a, y \rangle - \varepsilon\}$ $\{x : \langle a, x \rangle \leq \langle a, y \rangle\}$

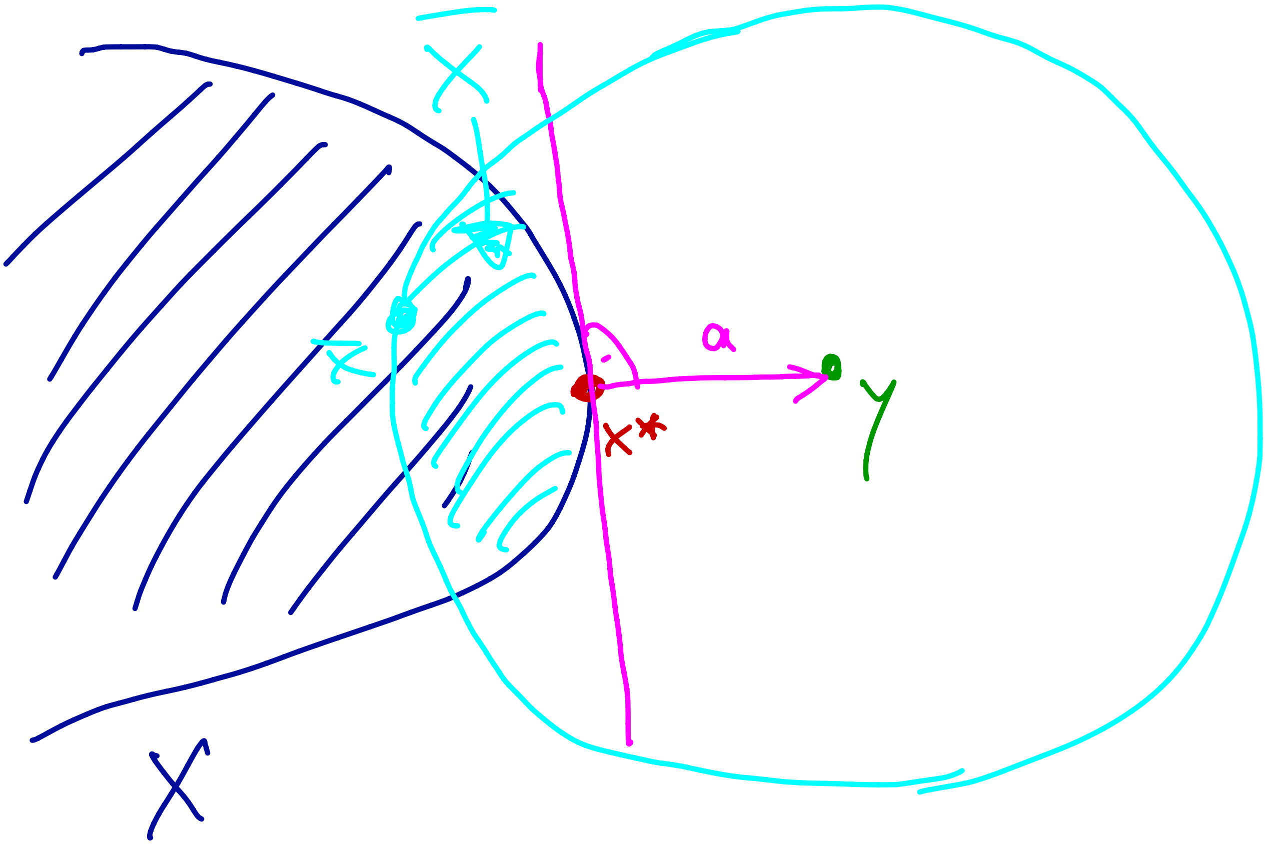


Konvexität!!!



Abgeschlossenheit!!!

Beweis zu Satz 2.15



- Können annehmen: $X \neq \emptyset$
  [sonst wähle a, ε beliebig]

- Sei $x^* \in X$ so, dass
  $\|y - x^*\| = \min \{ \|y - x\| : x \in X \}$.

  [Zur Existenz von x^*:
  - Sei $\bar{x} \in X \neq \emptyset$
  - $\bar{X} := \{ x \in X : \|y - x\| \leq \|y - \bar{x}\| \}$
  - $\inf \{ \|y - x\| : x \in X \} = \inf \{ \|y - x\| : x \in \bar{X} \}$

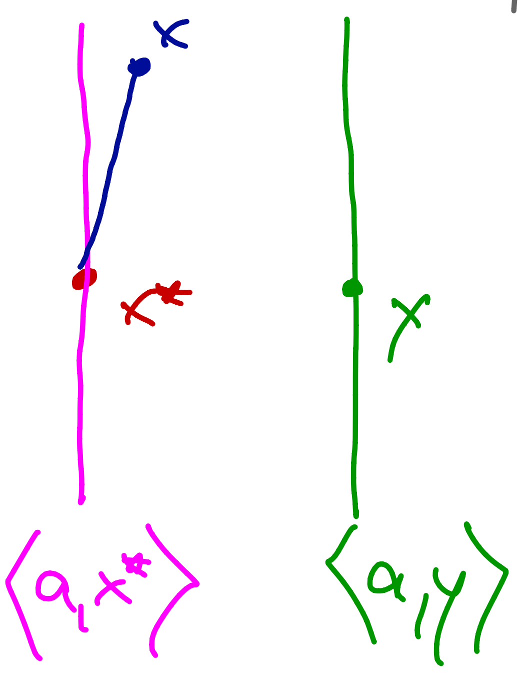
- Das Infimum auf der rechten Seite wird angenommen, weil X kompakt und $x \mapsto \|y-x\|$ stetig.]

- Setze $a := y - x^*$ und $\varepsilon := \|a\|^2$.

- Angenommen, es gäbe ein $x \in X$ mit

$$\langle a, x\rangle > \underbrace{\langle a, y\rangle - \varepsilon}_{\langle a,y\rangle - \langle a,a\rangle = \langle a, y-a\rangle = \langle a, x^*\rangle} \quad (**)$$

$(*)$



- Definiere $\varphi : \mathbb{R} \to \mathbb{R}$,

$$\varphi(t) := \|y - (x^* + t(x - x^*))\|^2$$

- Genügt, zu zeigen: $\varphi'(0) < 0$

[Denn dann gibt es $0 < \tilde{t} \leq 1$ mit

$$\|y-\tilde{x}\|^2 = \varphi(\tilde{t}) < \varphi(0) = \|y-x^*\|^2$$

mit $\tilde{x} = x^* + \tilde{t}(x-x^*)$.

- Weil X konvex und $x^*, x \in X$ (und $0 \leq \tilde{t} \leq 1$) ist $\tilde{x} \in X$ im Widerspruch zur Minimalität von x^*.]

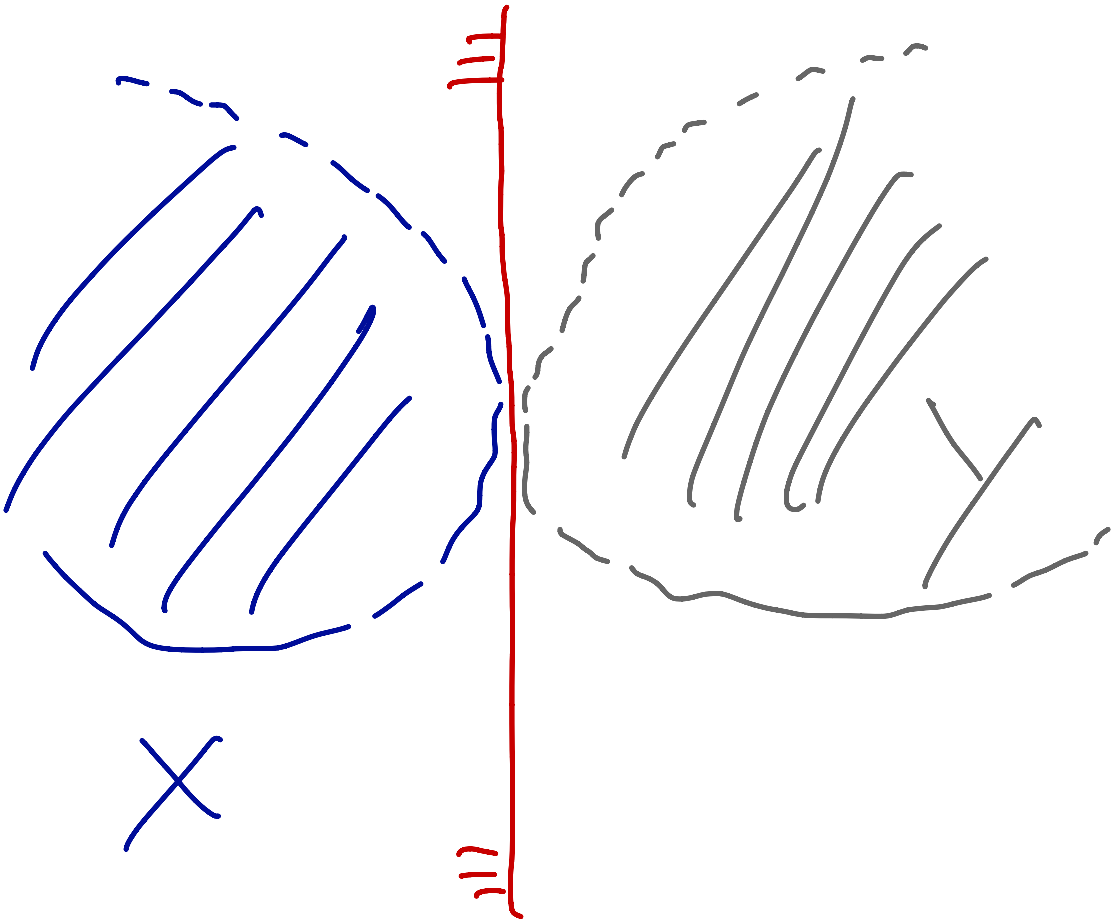
- $$\varphi'(t) = \frac{\partial}{\partial t} \langle a + t(x^*-x), a + t(x^*-x) \rangle$$

$$= \frac{\partial}{\partial t} \left(\langle a,a \rangle + 2\langle a, x^*-x \rangle t + \|x^*-x\|^2 \cdot t^2 \right)$$

$$= 2\langle a, x^*-x \rangle + 2\|x^*-x\|^2 \cdot t$$

- $$\varphi'(0) = 2\langle a, x^*-x \rangle$$

$$= 2 \Big(\underbrace{\langle a, x^* \rangle - \langle a, x \rangle}_{<0 \text{ wegen } (*)} \Big) \quad \blacksquare$$


X
Y