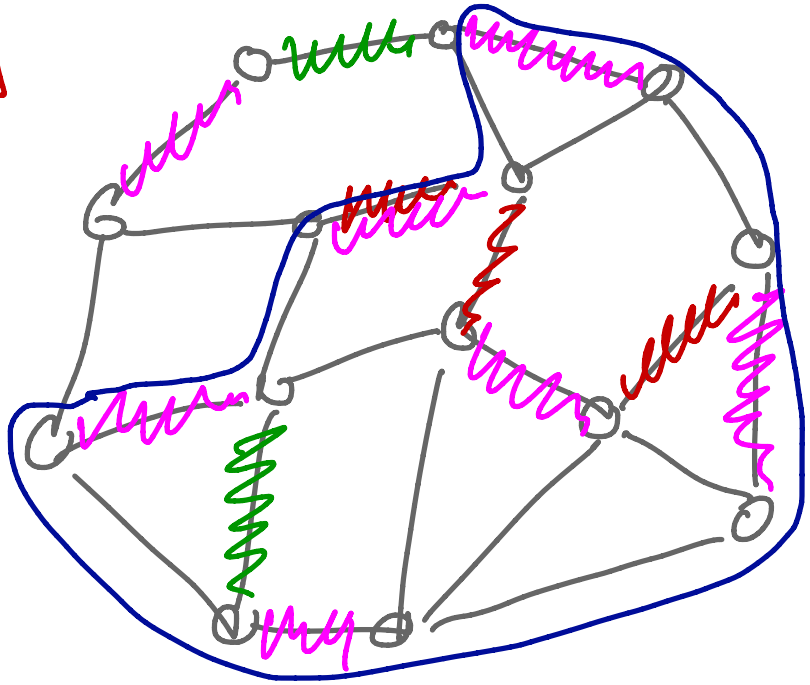
CAO 03.11.17

[30]



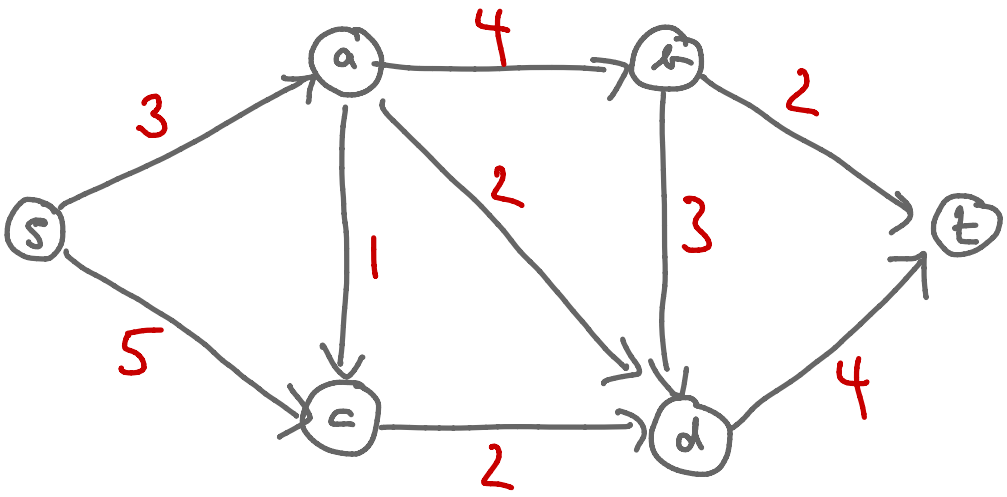
matching

not a matching

perfect matching

[33] Max-Flow-Problem

u : upper capacities



LP-model:

$$\max \quad x_{sa} + x_{sc}$$

"subject to" → s.t.

$$\begin{aligned}
x_{sa} - x_{ab} - x_{ad} - x_{ac} &= 0 \quad [\text{node } a] \\
x_{ab} - x_{bt} - x_{bd} &= 0 \quad [\text{node } b] \\
x_{ac} + x_{sc} - x_{cd} &= 0 \quad [\text{node } c] \\
x_{cd} + x_{ad} + x_{bd} - x_{dt} &= 0 \quad [\text{node } d]
\end{aligned}$$

$$\begin{aligned}
0 \le x_{sa} \le 3 \qquad & 0 \le x_{ad} \le 2 \\
0 \le x_{sc} \le 5 \qquad & 0 \le x_{cd} \le 2 \\
0 \le x_{ac} \le 1 \qquad & 0 \le x_{bt} \le 2 \\
0 \le x_{ab} \le 4 \qquad & 0 \le x_{dt} \le 4 \\
0 \le x_{bd} \le 3 &
\end{aligned}$$

Note:

$$x_{sa} - x_{ab} - x_{ad} - x_{ac} = 0$$

$$\Leftrightarrow \begin{pmatrix} x_{sa} - x_{ab} - x_{ad} - x_{ac} \leq 0 \\ -x_{sa} + x_{ab} + x_{ad} + x_{ac} \leq 0 \end{pmatrix} \quad \text{and}$$

General model for Max-Flow-Problem

$$\max \sum_{a \in \delta^{out}(s)} x_a \qquad \left(\text{assuming } \delta^{in}(s) = \emptyset\right)$$

$$\text{s.t.} \sum_{a \in \delta^{in}(v)} x_a - \sum_{a \in \delta^{out}(v)} x_a = 0 \quad \text{for all } v \neq s,t$$

$$0 \leq x \leq u$$

$x\left(\delta^{in}(v)\right)$