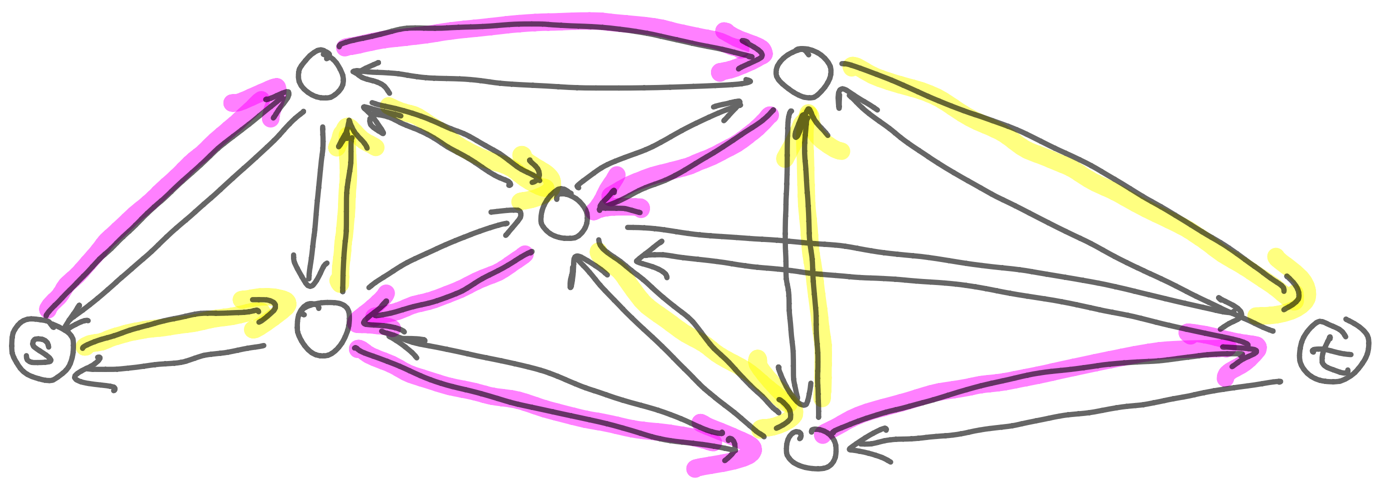
CAO 5.12.17

Directed s-t-Hamiltonian Path Problem

given: $D = (V, A)$ directed graph (complete)
$c(a) \in \mathbb{R}$ for all $a \in A$ (weights)
$s, t \in V$ $(s \neq t)$

task: Find an s-t-path in D that visits every node exactly once ("s-t-Hamiltonian path") of minimal weight w.r.t. the given weights.

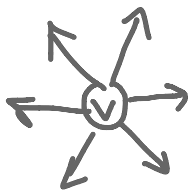


(figure does not show all arcs in the complete digraph D)

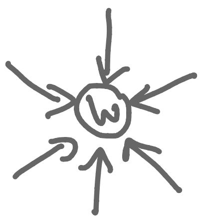
1st MIP - model for the problem

variables: $x_{(v,w)} \in \{0,1\} \quad \big((v,w) \in A\big)$

$$\min \sum_{(v,w)\in A} c(v,w) \cdot x_{(v,w)}$$



$$\sum_{w \in V \setminus \{v\}} x_{(v,w)} = 1 \qquad \forall v \in V \setminus \{t\}$$

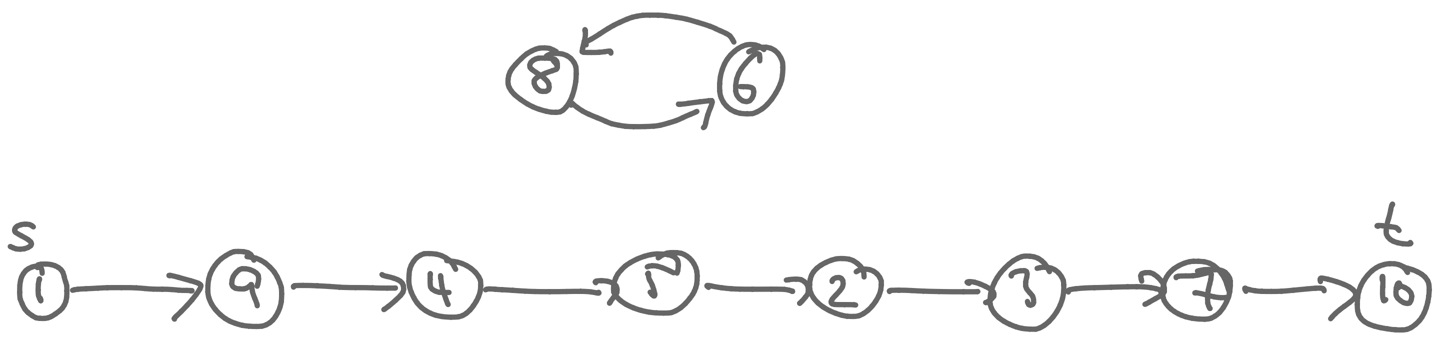


$$\sum_{v \in V \setminus \{w\}} x_{(v,w)} = 1 \qquad \forall w \in V \setminus \{s\}$$

$$\sum_{w \in V \setminus \{t\}} x_{(t,w)} = 0$$

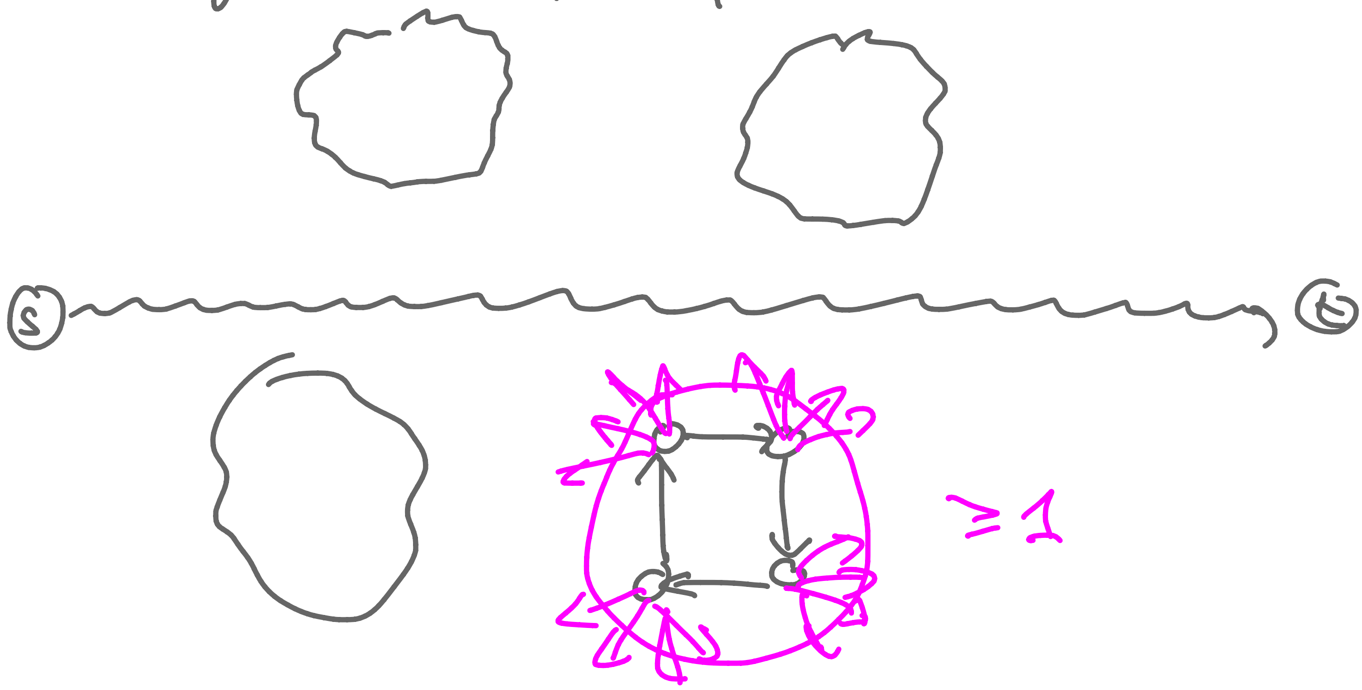
$$\sum_{v \in V \setminus \{s\}} x_{(v,s)} = 0$$

Solution from AMPL-model (1st attempt)



$$x_{(v,w)} + x_{(w,v)} \le 1 \qquad \forall\, v \neq w$$

More general difficulty:

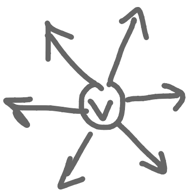


Correct(ed) MIP-model for the problem

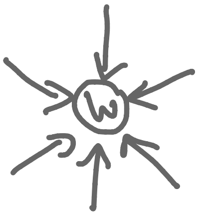
variables: $\quad x_{(v,w)} \in \{0,1\} \quad ((v,w) \in A)$

"position of in the path" $\quad p_v \in \{1,2,\dots,n\} \quad (v \in V)$

$$\min \sum_{(v,w)\in A} c(v,w) \cdot x_{(v,w)}$$



$$\sum_{w \in V\setminus\{v\}} x_{(v,w)} = 1 \qquad \forall v \in V\setminus\{t\}$$

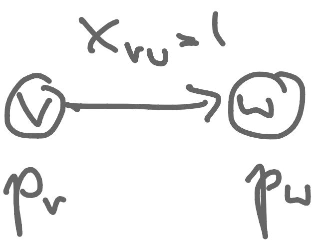


$$\sum_{v \in V\setminus\{w\}} x_{(v,w)} = 1 \qquad \forall w \in V\setminus\{s\}$$

$$\sum_{w \in V\setminus\{t\}} x_{(t,w)} = 0$$

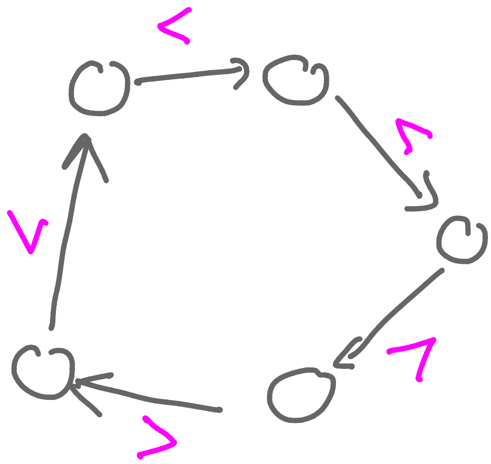
$$\sum_{v \in V\setminus\{s\}} x_{(v,s)} = 0$$

$$-n x_{vw} + p_w - p_v \geq -(n-1) \qquad \forall (v,w) \in A$$



"$x_{vw} = 1 \Rightarrow (p_w - p_v) \geq 1$"

Suppose $\{(v,w) \in A : x_{(v,w)} = 1\}$ contains a cycle:



p-values