CAO 12.12.17

Example of a Mixed Integer Set in $\mathbb{R}^2$

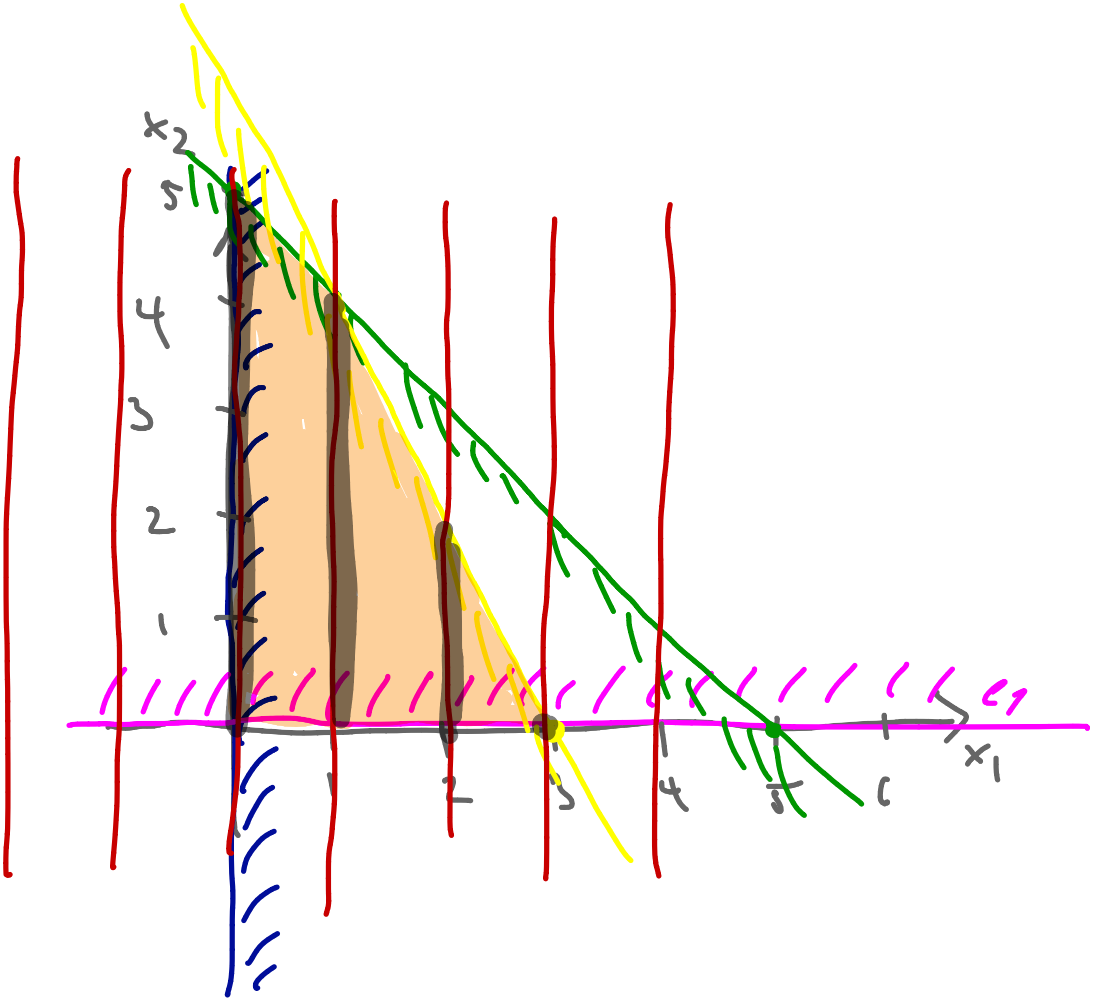
$x_1 \geq 0$

$x_2 \geq 0$

$x_1 + x_2 \leq 5$

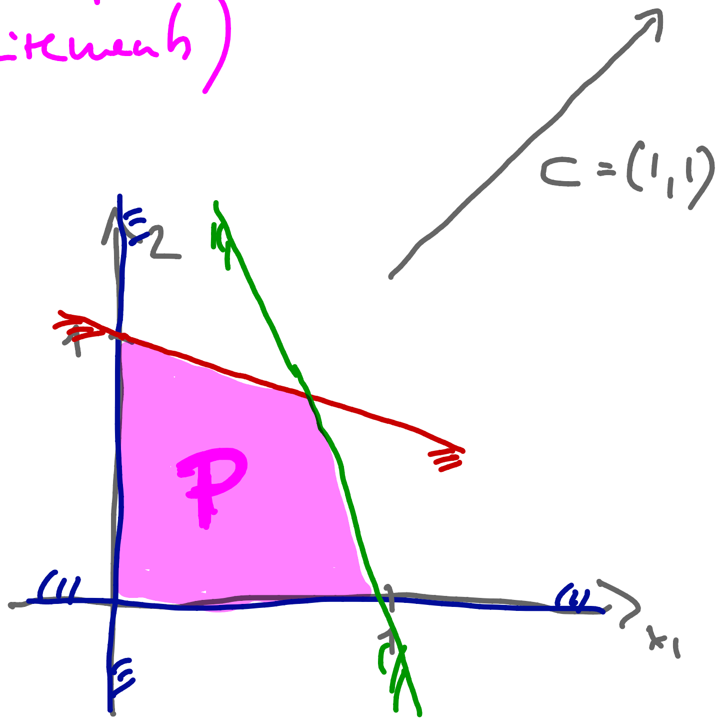
$2x_1 + x_2 \leq 6$

$x_1 \in \mathbb{Z}$



Example of the $\boxed{\text{Simplex-Algorithm}}$

Algorithm for solving Linear Programs (i.e. linear optimization problem without any integrality requirements)



$$
\begin{array}{ll}
\max & 1\cdot x_1 + 1\cdot x_2 \\
\text{s.t.} & 1\cdot x_1 + 3\cdot x_2 \leq 3 \\
 & 3\cdot x_1 + 1\cdot x_2 \leq 3 \\
 & x_1 \geq 0 \\
 & x_2 \geq 0
\end{array}
$$

Reformulation into LP-standard-form

$$
\begin{array}{ll}
\max & \xi \\
\text{s.t.} & x_1 + x_2 = \xi \\
 & x_1 + 3x_2 + x_3 = 3 \\
 & 3x_1 + x_2 + x_4 = 3 \\
 & x_1, x_2, x_3, x_4 \geq 0
\end{array}
$$

$$\boxed{\begin{aligned} x_1 + x_2 &= \xi \\ x_1 + 3x_2 + x_3 &= 3 \\ 3x_1 + x_2 + x_4 &= 3 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}}$$

$$\Longleftrightarrow \quad \boxed{\begin{aligned} x_3 &= 3 - x_1 - 3x_2 \\ x_4 &= 3 - 3x_1 - x_2 \\ \xi &= x_1 + x_2 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}} \quad (1)$$

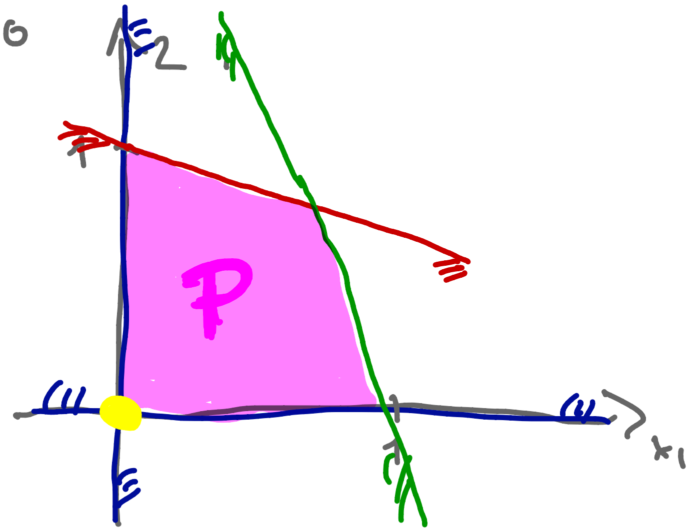
- Tableau (1): Representation of the "basic variables" x_3, x_4 as affine functions of the "non-basic variables" x_1, x_2.

- Task: Choose nonnegative values for the non-basic variables x_1, x_2, such that the basic variables x_3, x_4 are nonnegative and ξ is maximized.

• "Basic-solution for basis $\{3,4\}$":

$$\boxed{x_1^{(1)} = 0 \qquad x_2^{(1)} = 0} \qquad x_3^{(1)} = 3 \geq 0 \qquad x_4^{(1)} = 3 \geq 0 \qquad \xi^{(1)} = 0$$

"feasible basis"



• Observation: Increasing x_1 increases ξ

[because x_1 has a positive coefficient

in $\xi = \underline{1} \cdot x_1 + \underline{1} \cdot x_2$]

reduced costs w.r.t. current basis

• Step: In order to obtain $x^{(2)}$, we set $x_1^{(2)}$ to the largest positive value $\delta \geq 0$ with

$$x_3^{(2)} = 3 - 1 \cdot \delta \geq 0 \iff \delta \leq \frac{3}{|-1|}$$

$$x_4^{(2)} = 3 - 3 \cdot \delta \geq 0 \iff \delta \leq \frac{3}{|-3|}$$

"ratio test"

thus $\delta := 1$.

We obtain the solution

$$\begin{array}{ccccc} x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & x_4^{(2)} & \xi^{(2)} \\ \shortparallel & \shortparallel & \shortparallel & \shortparallel & \shortparallel \\ 1 & 0 & 2 & 0 & 1 \end{array}$$

- $x^{(2)}$ is the basic-solution associated with the new basis $\{1,3\}$, which has been obtained from the old basis $\{3,4\}$ by exchanging x_4 for x_1.

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