Definition. *A* ***network*** *is a digraph D along with some* ***lower / upper capacity vectors***

$$\ell \in \mathbb{R}^{\mathrm{A}(D)} \quad / \quad u \in \mathbb{R}^{\mathrm{A}(D)}$$

on the arcs with $\ell \le u$ (i.e., $\ell_a \le u_a$ for all $a \in \mathrm{A}(D)$); the lower capacities are assumed to be $\ell = \mathbb{O}$, in case they are not specified.

[9]

Definition. *Let D be a digraph. For $v \in \mathrm{V}(D)$ we call*

$$\delta^{\text{out}}(v) := \delta^{\text{out}}_D(v) := \mathrm{A}(D) \cap (\{v\} \times \mathrm{V}(D))$$

the ***out star*** *of v and*

$$\delta^{\text{in}}(v) := \delta^{\text{in}}_D(v) := \mathrm{A}(D) \cap (\mathrm{V}(D) \times \{v\})$$

the ***in star*** *of v.*

[10]

Definition. *The* ***transportation problem*** *is for a given network D with upper capacity vector $u \in \mathbb{R}^{\mathrm{A}(D)}_+$, $\mathrm{V}(D) = S \cup T$ ($S \cap T = \varnothing$) and $\mathrm{A}(D) \subseteq S \times T$, a supply vector $\sigma \in \mathbb{R}^S$, a demand vector $\delta \in \mathbb{R}^T$, and a cost vector $c \in \mathbb{R}^{\mathrm{A}(D)}$ to find some flow vector $f \in \mathbb{R}^{\mathrm{A}(D)}_+$ with $f \le u$ and*

- $f(\delta^{\text{out}}(s)) = \sigma_s$ *for all* $s \in S$ *and*
- $f(\delta^{\text{in}}(t)) = \delta_t$ *for all* $t \in T$

with minimal costs $c(F)$.

[11]

Definition. *The **max flow problem** is for a given network D with upper capacity vector $u \in \mathbb{R}_+^{\mathrm{A}(D)}$ and two given nodes $s, t \in \mathrm{V}(D)$ to find a flow vector $f \in \mathbb{R}_+^{\mathrm{A}(D)}$ with $f \le u$ and*

$$f(\delta^{\text{in}}(v)) = f(\delta^{\text{out}}(v))$$

*for all $v \in \mathrm{V}(D) \setminus \{s, t\}$ (such an f is called an s-t-**flow**) with maximal flow value*

$$f(\delta^{\text{out}}(s)) - f(\delta^{\text{in}}(s))$$

(which equals $f(\delta^{\text{in}}(t)) - f(\delta^{\text{out}}(t))$).

[12]

Definition. *The **min cost circulation problem** is for a given network D with lower and upper capacity vectors $\ell, u \in \mathbb{R}^{\mathrm{A}(D)}$ and a cost vector $c \in \mathbb{R}^{\mathrm{A}(D)}$ to find a flow vector $f \in \mathbb{R}^{\mathrm{A}(D)}$ with $\ell \le f \le u$ and*

$$f(\delta^{\text{in}}(v)) = f(\delta^{\text{out}}(v))$$

*for all $v \in \mathrm{V}(D)$ (such an f is called a **circulation**) with maximal costs $c(f)$.*

[13]