Definition. *A subset $M \subseteq \mathrm{E}(G)$ of the edges of a graph G is called a* ***matching*** *in G, if $e \cap e' = \varnothing$ holds for all $e, e' \in M$ with $e \neq e'$; a matching M is called* ***perfect****, if $|M| = \frac{1}{2}|\mathrm{V}(G)|$ holds (i.e., every node of T is in one edge of the matching).*

[30]

Definition. *The* ***matching problem*** *is to find for a given graph G and a weight vector $w \in \mathbb{R}^{\mathrm{V}(G)}$ a matching $M \subseteq \mathrm{E}(G)$ with maximal weight $w(M)$. The matching problem for bipartite graphs is called the* ***assignment problem****.*

[31]

Chapter 2

Mixed Integer Linear Optimization

Mixed Integer Linear Optimization
=
Mixed Integer Linear Programming (**MILP**)

Input:

$$A \in \mathbb{Q}^{m \times n}, b \in \mathbb{Q}^m, c \in \mathbb{Q}^n, I \subseteq [n]$$

Feasible solutions:

$$X := \{x \in \mathbb{R}^n \ : \ Ax \leq b, x_i \in \mathbb{Z} \text{ for all } i \in I\}$$

Task: Find *optimal solution* $x^{\text{opt}} \in X$ with

$$\langle c, x^{\text{opt}} \rangle = \max\{\langle c, x \rangle \ : \ x \in X\}$$

[32]

$I = \varnothing$:

Linear Optimization
=
Linear Programming (**LP**)

$I = [n]$:

Integer Linear Optimization
=
Integer Linear Programming (**ILP**)

[33]