



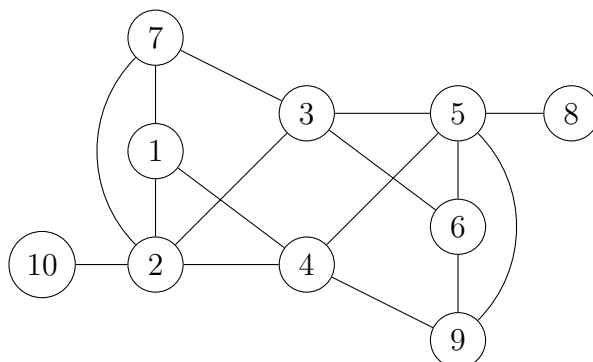
## Concepts and Algorithms of Optimization – Series 3

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### Exercise 1

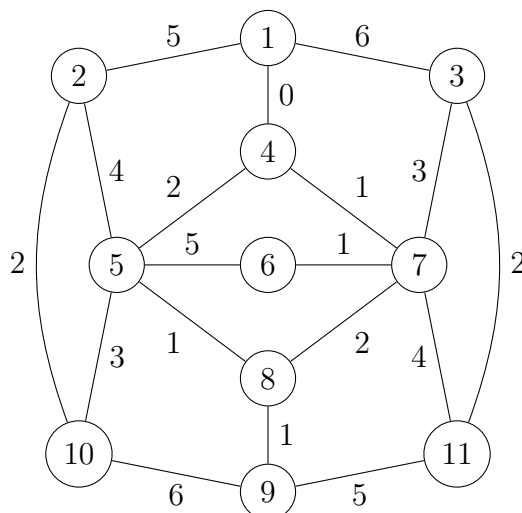
Consider the following undirected graph  $G = (V, E)$ .



- Determine two subgraphs  $G' = (V', E')$  and  $G'' = (V'', E'')$  induced by the node sets  $V' = \{1, 2, 3, 4, 7\}$  and  $V'' = \{2, 5, 6, 8, 9, 10\}$ , respectively.
- State whether the given subgraphs  $G'$  and  $G''$  have the following properties:
  - The subgraph is spanning.
  - The subgraph is connected.
  - The subgraph is cyclic.
  - The subgraph has exactly one component.
- Determine a spanning subgraph  $G^F$  of  $G$ , which is a forest but no tree. Give the necessary changes in  $G^F$ , so that it becomes a tree.

### Exercise 2

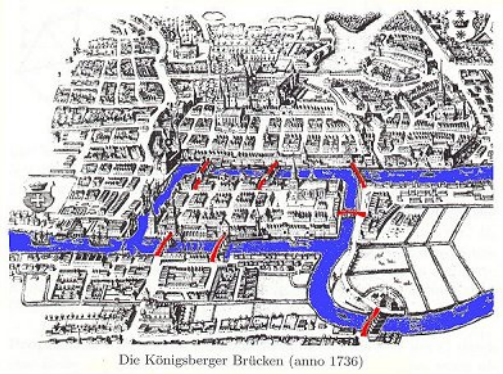
Consider the following graph  $G = (V, E)$  with given edge costs  $c_e$  for all  $e \in E$ .



- (a) Determine a spanning tree  $T$  in  $G$  with minimal costs  $c(T)$ .
- (b) Give a general description of the algorithm applied in (a).
- (c) Determine all cliques in  $G$ .
- (d) Find a stable set  $S^*$  in  $G$  with maximal weight  $w(S^*)$  regarding the node weights  $w_v = 1$  for all  $v \in V$ .  
(Hint: This corresponds to finding a stable set with maximal cardinality in  $G$ .)

### Exercise 3

In the early 18th century there was a little boy called James living in Königsberg. The city is split up into two parts by the Pregel river and there exist two islands within the river. The people in Königsberg had built seven bridges to connect the two parts of the city and the islands. One day James had a walk with his father to a small church on a hill nearby. Looking at the city with its beautiful bridges, he asked his father: Could we have a walk by which we cross every bridge exactly once?



- (a) Give a graph-based representation of the city and a corresponding formulation of James' question.
- (b) Answer James' question and give a theoretic argument.