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Concepts and Algorithms of Optimization – Series 1

www.math.uni-magdeburg.de/institute/imo/teaching/wise18/cao/

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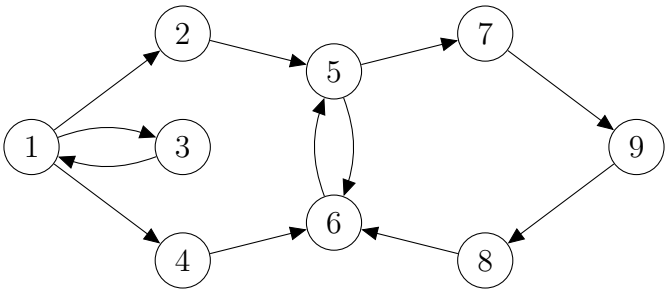
Exercise 1

A Digraph $D = (V, A)$ is given as follows:



(a) Determine all out-arcs and all in-arcs of the nodes $v = 3$ and $w = 6$.

(b) Determine all out-neighbors and all in-neighbors of node $v = 5$.

(c) Give the node set, the arc set and the length of two different 1-8-paths P_1 and P_2 in D, respectively.

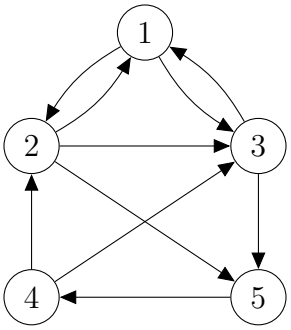
(d) Determine a cycle C_1 with length $k_1 = 5$ and a cycle C_2 with length $k_2 = 2$ in D.

(e) Give a subdigraph D' and a spanning subdigraph D'' of D.

(f) Does there exist a Hamiltonian path or a Hamiltonian cycle in D?

Exercise 2

Consider the following digraph $D = (V, A)$.



(a) Determine a Hamiltonian path P_H and a Hamiltonian cycle C_H in D.

(b) Give the out-star $\delta^{out}(v)$ and the in-star $\delta^{in}(v)$ of node $v = 3$.

Exercise 3

The company *Color Ink* spray-paints wooden toys for children in 7 different colors, namely: white, yellow, red, green, blue, purple and black.
The spray-painting is done by one machine, which has to be cleaned after every painting process. The intensity and duration of the cleaning procedure depend on the colors used to paint the preceding and the succeeding toy. Especially, there are several color sequences that are not allowed to be processed. The cleaning time t is given in the following table for each color sequence whereby '-' denotes an infeasible order.

t in *min*	*successor*						
predecessor	white	yellow	red	green	blue	purple	black
white	0	0	-	0	-	-	-
yellow	10	0	30	20	-	-	0
red	-	30	0	-	10	20	0
green	-	10	-	0	10	10	0
blue	-	-	-	-	0	10	0
purple	-	-	10	-	10	0	0
black	-	-	-	-	-	-	0

For the next day, there are the following 8 toys to be painted: a snowman (white), a flower (yellow), an apple (red), two trees (green), a fish (blue), a puppet (purple) and a car (black). The production manager of the company is willing to find the coloring sequence of all toys with minimum total cleaning time.

(a) Transfer the sequencing problem of the *Color Ink* company to a directed graph. Which underlying graph-based problem is to be solved by the production manager?

(b) Give two different feasible sequences for the toy coloring problem and the corresponding total cleaning time.