

## Concepts and Algorithms of Optimization – Series 2

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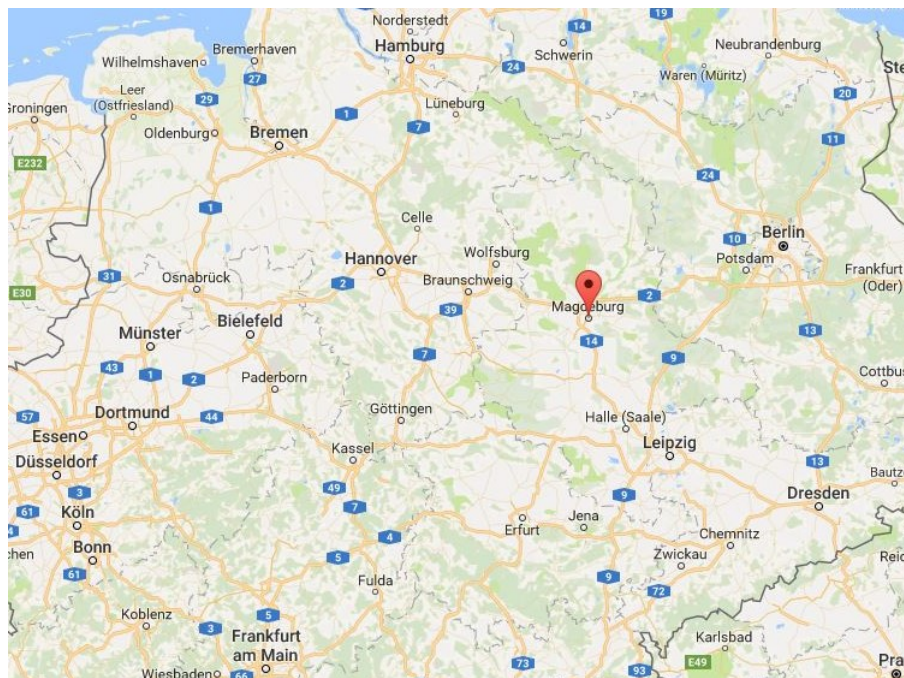
October 19, 2018

### Exercise 1

A construction manager is working on several projects in different cities. His office is located in Magdeburg (MD) and he is planning to visit construction sites in Berlin (B), Leipzig (L), Hannover (H), Hamburg (HH) and Frankfurt (F) next week.

His goal is to find a tour through all of the five cities starting and ending in Magdeburg and including the least amount of kilometers. From experience, it is not reasonable to have a direct trip between Hannover and Berlin in the tour and he is not willing to visit the construction site in Frankfurt as the first or last one. The table given below states the travel distances between all pairs of cities.

in <i>km</i>	B	L	H	HH	F
MD	155	130	150	260	430
B	-	190	290	290	550
L		-	265	400	400
H			-	155	350
HH				-	500



- Give a graph-based problem formulation to determine the shortest tour for the construction manager.
- State two different feasible tours and their total lengths in *km*.

**Exercise 2**

The *Best Bread* company is willing to analyze the implemented bread production and transportation strategy. The company operates three bakeries with given production levels in Berlin (B, 800 breads per day), Hamburg (HH, 1000 breads per day) and Hannover (H, 1200 breads per day).

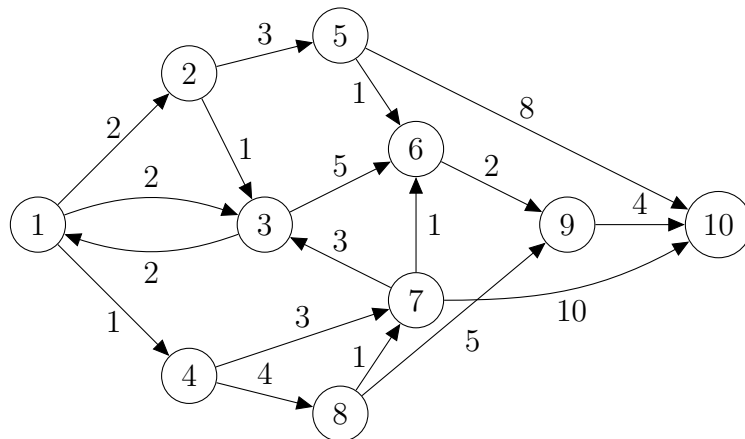
The customers are located in Berlin (B), Osnabrück (O), Bielefeld (BF), Göttingen (G), Bremen (HB), Magdeburg (MD), Lüneburg (HL), Leipzig (L), Dresden (DD) and Schwerin (S). Following the transportation policy of the last years, the *Best Bread* company has already assigned small trucks (maximum capacity: 200 breads) and large trucks (maximum capacity: 500 breads) to the established routes. Furthermore, a longterm supplier contract requires a minimum amount of 100 breads to be delivered from Hamburg to Magdeburg. The following table provides the total demand per city as well as the truck assignment (s/l) and the costs (in Euro per bread) for the existing routes.

	B	O	BF	G	HB	MD	HL	L	DD	S
<i>demand</i>	100	350	200	400	400	300	250	200	400	400
B	s (1)	-	-	-	-	s (6)	-	s (4)	l (10)	-
HH	-	-	-	-	l (2)	s (8)	l (4)	-	-	l (6)
H	-	l (10)	s (2)	l (4)	s (6)	s (4)	-	-	-	-

- (a) By means of a graph-based representation, determine the amount of delivered breads on each route causing the minimum costs for the *Best Bread* company.
- (b) Show how this transportation problem can be formulated as a min cost circulation problem.

**Exercise 3**

Consider the following digraph  $D = (V, A)$  with given costs  $c_a$  for  $a \in A$ .



- (a) Give a problem formulation for finding the cost-minimal path between the nodes  $s = 1$  and  $t = 10$  in  $D$ .
- (b) Transform the given problem into a min cost circulation problem.