

7.3 Standardintegrale

$$1. \int x^r dx = \frac{x^{r+1}}{r+1} + c, \quad r \in \mathbb{R}, \quad r \neq -1$$

$$\int (ax+b)^r dx = \frac{(ax+b)^{r+1}}{a(r+1)} + c, \quad r \in \mathbb{R}, \quad r \neq -1$$

Beispiele:

$$\int \frac{dx}{(x-2)^2} = \int (x-2)^{-2} dx = \frac{(x-2)^{-1}}{-1} + c = \frac{-1}{(x-2)} + c$$

$$\int \frac{1}{\sqrt[3]{4x+7}} dx = \int (4x+7)^{-1/3} dx = \frac{(4x+7)^{2/3}}{4 \cdot \frac{2}{3}} + c$$

$$2. \int \frac{1}{x} dx = \ln|x| + c, \quad x \neq 0$$

$$\int \frac{1}{x+a} dx = \ln|x+a| + c, \quad x \neq -a$$

$$3. \int \sin ax dx = -\frac{1}{a} \cos ax + c$$

$$4. \int \cos ax dx = \frac{1}{a} \sin ax + c$$

$$5. \int \frac{1}{\cos^2 ax} dx = \frac{1}{a} \tan ax + c$$

$$6. \int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$7. \int a^x dx = \frac{a^x}{\ln a} + c, \quad a \neq 1, \quad a > 0$$

$$8. \int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} + c_1 = -\arccos \frac{x}{a} + c_2, \quad |x| < a, \quad c_2 = c_1 + \frac{\pi}{2}$$

$$9. \int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + c$$

$$10. \int \sinh(ax) dx = \frac{1}{a} \cosh(ax) + c$$

$$11. \int \cosh(ax) dx = \frac{1}{a} \sinh(ax) + c$$

$$12. \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2}) + c_1 = \operatorname{arsinh}\left(\frac{x}{a}\right) + c_2, \quad c_2 = c_1 - \ln|a|$$

$$13. \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln(x + \sqrt{x^2 - a^2}) + c_1 = \operatorname{arcosh}\left(\frac{x}{a}\right) + c_2, \quad c_2 = c_1 - \ln|a|$$

$$14. \int \frac{dx}{a^2 - x^2} = \left\{ \begin{array}{l} \frac{1}{a} \operatorname{artanh}\left(\frac{x}{a}\right) + c, \quad |x| < a \\ \frac{1}{a} \operatorname{arcoth}\left(\frac{x}{a}\right) + c, \quad |x| > a \end{array} \right\} = \frac{1}{2} \ln \left| \frac{a+x}{a-x} \right| + c, \quad |x| \neq a$$

$$15. \int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

Beispiel:

$$\int \tan 3x dx = -\frac{1}{3} \int \frac{-3 \sin 3x}{\cos 3x} dx = -\frac{1}{3} \ln|\cos 3x| + c$$