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Tutorial Stochastic Processes
(Serie 1, 2011/12 Basics)

1. The random variable X is defined by $P(X = -2) = 1/3$, $P(X = 1) = 1/2$ and $P(X = 2) = 1/6$. Calculate the distribution function $F_X(x)$, the expectation $E(X)$ and the variance $\mathrm{Var}(X)$.

2. Let Y be the face value on a fair die. Calculate the expectation $E(Y)$ and the variance $\mathrm{Var}(Y)$.

3. Let U be the continuous uniform r.v. on the interval $(2\,,\,6)$. Calculate the distribution function $F_U(x)$, the expectation $E(U)$ and the variance $\mathrm{Var}(U)$.

4. Let $X \sim Exp(\lambda)$. Prove that the exponential distributed r.v. X is memoryless, i.e. $P(X > x + y | X > x) = P(X > y)$ for all $x > 0$ and $y > 0$, where $P(A|B) = \frac{P(A \cap B)}{P(B)}$ denotes the conditional probability.

5. Let $X = (X_1, X_2, X_3)$ be a random vector, where X_1, X_2, X_3 are independent and the r. v. X_i are $N(i\,,\,i^2)$-distributed, $i = 1, 2, 3$.

   Define $Y_1 = X_1 - X_2$, $Y_2 = X_3 - X_1$, $Y_3 = X_1$ and $Y = (Y_1, Y_2, Y_3)$.

   Find the expectation vectors $E(X)$ and $E(Y)$ and covariance-matrices Σ_X and Σ_Y !

References to Stochastic Processes

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