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Tutorial Stochastic Processes
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5.) Let X_i be $N(i\,,\,i^2)$-distributed, $i = 1, 2, 3$, and let X_1, X_2, X_3 be independent. Define $Y_1 = X_1 - X_2$, $Y_2 = X_3 - X_1$ and $Y_3 = X_1$.

Find the expectation vectors and covariance-matrices $E(X_1, X_2, X_3)$, $\Sigma_{(X_1,X_2,X_3)}$, $E(Y_1, Y_2, Y_3)$ and $\Sigma_{(Y_1,Y_2,Y_3)}$!

6.) In Ex 1.8 the two-dimensional Gaussian density was defined as

$$f_{(X_1,X_2)}(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}$$
$$\times \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 - 2\rho\frac{(x_1-\mu_1)}{\sigma_1}\frac{(x_2-\mu_2)}{\sigma_2} + \left(\frac{x_2-\mu_2}{\sigma_2}\right)^2\right]\right\} .$$

Show that it coincides with formula (2.1) of the n-dimensional Gaussian density in case $n = 2$.

7.) Let $X_1, X_2, ...$ be independent identically distributed (i.i.d.) random variables. Consider the random walk $\{S_n\}_{n\geq 0}$, given by $S_0 = 0, S_k = X_1 + ... + X_k$, $1 \leq k \leq n$. Verify that S_n has stationary, independent increments (Ex.2.1). In case of the random walk with $P(X_k = \pm 1) = 1/2$ (Ex.2.1) find for the vector $(S_1, S_2, ..., S_n)$ the covariance matrix $\Sigma_{(S_1,...S_n)}$ and for the process $\{S_n\}_{n\geq 0}$ the expectation function $m(k) = E(S_k)$, $k = 0, 1, ..., n$ and the covariance function $C_S(k, m) = Cov(S_k, S_m)\,,\ 1 \leq k, m \leq n$!

8.) Let N_T be a homogeneous Poisson process with parameter $\lambda > 0$. Calculate the expectation function $m_N(t) = E\,N(t)$, the covariance-function $Cov(s, t)$ and $E(N(t)\,N(s))$.

9.) Let N_T be a homogeneous Poisson process with parameter $\lambda > 0$. Calculate for $t_1 < t_2$
a) $P(N(t_2) = k_2 \,|\, N(t_1) = k_1)\,,$
b) $P(N(t_1) = k_1 \,|\, N(t_2) = k_2)$.

10.) Let $\{X(t);\, t \geq 0\}$ and $\{Y(t);\, t \geq 0\}$ be independent homogeneous Poisson processes with parameters λ_1 and λ_2, respectively. Define
a) $Z_1(t) = X(t) + Y(t)$,
b) $Z_2(t) = X(t) - Y(t)$ and
c) $Z_3(t) = X(t) + k$ (Here $k \geq 0$ is a fixed integer).
Which of the processes are such with independent increments, which of them are Poisson processes?