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Tutorial Stochastic Processes (Series 3, 2011/2012)

11.) Let $N_t, t \geq 0$ be a homogeneous Poisson process with intensity $\lambda > 0$ and τ the time till the first event. For $s < t$ find $P(\tau \leq s | N(t) = 1)$!

12.) Suppose that $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$ are independent Poisson processes with rates λ_1 and λ_2. $\{N_1(t) + N_2(t), t \geq 0\}$ is a Poisson process with rate $\lambda_1 + \lambda_2$. (Show it!). Prove that the probability that the first event of the combined process $\{N_1(t) + N_2(t), t \geq 0\}$ comes from the first one $\{N_1(t), t \geq 0\}$ is $\lambda_1 \,/\, (\lambda_1 + \lambda_2)\,$!

13.) Let $W(t)$ be a standard Wiener process. Using the fact, that a Gaussian process is determined by its expectation and covariance functions, prove: the following processes are standard Wiener processes, too
$W_1(t) = -W(t),\ t \in [0, \infty)\,,$

$W_2(t) = c \cdot W(t \cdot c^{-2}),\ c > 0,\ t \geq 0\,,$

$W_3(t) = t \cdot W(1/t)$ for $t > 0,\ (W_3(0) = 0),\ t \in [0, \infty)$ and

$W_4(t) = W(h - t) - W(h),\ h$ fixed, $t \in [0, h]$.

14.) Let $W(t)$ be a Wiener process, $L > 0$ a constant and $A \sim N(\mu, \tau^2)$ be a random variable independent of $\{W(t)\}_{t \in [0,\infty)}$. Calculate mean vector and covariance function of the vectors $(X_k(s), X_k(t)),\ s < t$, where

a) $X_1(t) = W(t + L) - W(L)$, b) $X_2(t) = W(t + L) - W(t)$

c) $X_3(t) = A \cdot t + W(t)$, d) $X_4(t) = \begin{cases} (1-t) W\left(\frac{t}{1-t}\right) & , \quad 0 \leq t < 1 \\ 0 & , \quad t \geq 1\,. \end{cases}$

15.) Let $(W_t,\, t \geq 0)$ be a standard Wiener Prozess, then $B(t) = W_t - t\, W_1$ for $0 \leq t \leq 1$ is a *Brownian bridge.*

a) Prove $E\, B(t) = 0$ and $Cov_B(t, s) = \min\{t, s\} \; - \; t\, s$.

b) Prove, that the Process $X_t = (1 + t)\, B(\frac{t}{1+t})$ for $0 \leq t < \infty$ is a Wiener process, too.