Fakultät für Mathematik, Institut für Mathematische Stochastik
Prof. G. Christoph

Tutorial Stochastic Processes
(Serie 2, 2013/14)

6. Example 2.3: Let $X_1, X_2, ...$ be independent identically distributed (i.i.d.) random variables and $S_0 = 0, S_k = X_1 + ... + X_k,\ k \geq 1$, $k = 1, 2, .., n$. The sequence $\{S_n\}_{n\geq 0}$ is also called a random walk, too. Show that the random process $\{S_n\}_{n\geq 0}$ has stationary and independent increments.

7. In Ex 1.8 the two-dimensional Gaussian density was defined as

$$f_{(X_1,X_2)}(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}$$

$$\times \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 - 2\rho\frac{(x_1-\mu_1)}{\sigma_1}\frac{(x_2-\mu_2)}{\sigma_2} + \left(\frac{x_2-\mu_2}{\sigma_2}\right)^2\right]\right\} .$$

Show that it coincides with formula (2.1) of the n-dimensional Gaussian density in case $n = 2$.

8. Let N_T be a homogeneous Poisson process with parameter $\lambda > 0$. Calculate the expectation function $m_N(t) = E\,N(t)$, the covariance-function $Cov_N(s, t)$ and $E(N(t)\,N(s))$.

9. Let N_T be a homogeneous Poisson process with parameter $\lambda > 0$. Calculate for $t_1 < t_2$
   a) $P(N(t_2) = k_2 \,|\, N(t_1) = k_1)$,
   b) $P(N(t_1) = k_1 \,|\, N(t_2) = k_2)$.

10. Let $N_t, t \geq 0$ be a homogeneous Poisson process with intensity $\lambda > 0$ and τ the time till the first event. For $s < t$ find $P(\tau \leq s | N(t) = 1)$!

11. Suppose that $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$ are independent Poisson processes with rates λ_1 and λ_2. $\{N_1(t) + N_2(t), t \geq 0\}$ is a Poisson process with rate $\lambda_1 + \lambda_2$. (Show it!). Prove that the probability that the first event of the combined process $\{N_1(t) + N_2(t), t \geq 0\}$ comes from the first one $\{N_1(t), t \geq 0\}$ is $\lambda_1 / (\lambda_1 + \lambda_2)$!