Fakultät für Mathematik, Institut für Mathematische Stochastik
Prof. G. Christoph

Tutorial Stochastic Processes
(Serie 1, 2013/14 Basics)

1. The random variable X is defined by $P(X = -2) = 1/3$, $P(X = 1) = 1/2$ and $P(X = 2) = 1/6$. Calculate the distribution function $F_X(x)$, the expectation $E(X)$ and the variance $\mathrm{Var}(X)$.

2. Let Y be the face value on a fair die. Calculate the expectation $E(Y)$ and the variance $\mathrm{Var}(Y)$.

3. Let U be the continuous uniform r.v. on the interval $(2\,,\,6)$. Calculate the distribution function $F_U(x)$, the expectation $E(U)$ and the variance $\mathrm{Var}(U)$.

4. Let $X \sim Exp(\lambda)$. Prove that the exponential distributed r.v. X is memoryless, i.e. $P(X > x + y | X > x) = P(X > y)$ for all $x > 0$ and $y > 0$, where $P(A|B) = \frac{P(A \cap B)}{P(B)}$ denotes the conditional probability.

5. Let X_i be $N(i\,,\,i^2)$-distributed, $i = 1, 2, 3$, and let X_1, X_2, X_3 be independent.

   Define $Y_1 = X_1 - X_2$, $Y_2 = X_3 - X_1$ and $Y_3 = X_1$.

   Find the expectation vectors and covariance-matrices $E(X_1, X_2, X_3)$, $\Sigma_{(X_1,X_2,X_3)}$, $E(Y_1, Y_2, Y_3)$ and $\Sigma_{(Y_1,Y_2,Y_3)}$!

References to Stochastic Processes

1. Mikosch, Thomas: *Elementary Stochastic Calculus with Finance in View.* World Scientific, 2000.

2. Sondermann *Introduction to Stochastic Calculus for Finance. A New Didactic Approach.* Springer, 2006.

3. Steland, Ansgar: Financial Statistics and Mathematical Finance: Methods, Models and Applications. John Wiley & Sons.

4. Klebaner *Introduction to stochastic calculus with applications.* 2.ed. - London : Imperial College Press, 2005

5. Rozanov, Yu. A.: *Probability theory, random processes and mathematical statistics.* Kluwer, 1995 (engl., russian)

6. Ross, S.M.: *Stochastic processes.* Wiley, New York, 1996.

7. Grimmett, G.R., Stirzaker, D.R.: *Probability and Random Processes*, Oxford Science Publ., 1995