# Approaches for planning the ISS cosmonaut training * 

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#### Abstract

We consider the problem of planning the ISS cosmonaut training with different objectives. A pre-defined set of minimum qualification levels should be distributed between the crew members with minimum training time differences, training expenses or a maximum of the training level with a limitation of the budget.

First, a description of the cosmonaut training process is given. Then four models are considered for the volume planning problem. The objective of the first model is to minimize the differences between the total time of the preparation of all crew members, the objective of the second one is to minimize the training expenses with a limitation of the training level, and the objective of the third one is to maximize the training level with a limited budget. The fourth model considers the problem as an $n$-partition problem. Then two models are considered for the calendar planning problem.


For the volume planning problem, two algorithms are presented. The first one is a heuristic with a complexity of $O(n)$ operations. The second one consists of a heuristic and exact parts, and it is based on the $n$-partition problem approach.
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## 1 Relevance of the problem



Figure 1: ISS logotype

In Fig. 1, one can see the logotype of the International Space Station.

The International Space Station (ISS) is an artificial satellite, on a low Earth orbit. It is made of many modules. The first one was launched in 1998. Nowadays it is the biggest artificial object on an orbit, often it could be seen from the earth with a naked eye.

The ISS has the potential to conduct a wide spectrum of scientific researches. The experiments could last for decades with the possibility of a careful control of the humans. The ISS maintains an orbit with an altitude between
$330 \mathrm{~km}(205 \mathrm{mi})$ and $435 \mathrm{~km}(270 \mathrm{mi})$ by means of reboot maneuvers using the engines of the Zvezda module or the visiting spacecraft. It completes 15.51 orbits per day. The length is $72.8 \mathrm{~m}(239 \mathrm{ft})$, and the width is 108.5 m ( 356 ft ).

In Fig. 2, one can see the ISS when it would be placed on the Red square.


Figure 2: ISS in comparison with the Red square
Among all the cosmonautic problems, particular attention is dedicated to the planning problems. For scheduling the operations during the flight and for scheduling the trainings before, it is necessary to maximize the efficiency. Due to the date, it takes lots of human, time and material resources.

The proper preparation of cosmonauts is a long, expensive and sophisticated process. In order to maintain reliability of a flight, the crew members are obligated to be trained for different types of situations and operations,
to obtain required skills and knowledge before the launch. Hence, the Yu. A. Gagarin Research \& Test Cosmonaut Training Center (CTC) must plan and schedule a list of trainings for every cosmonaut.

In general, three crew qualification levels are defined; a user level, an operator level and a specialist level. For a given flight program, for every onboard complex, a pre-defined set of minimum qualifications is needed to safely operate and maintain the system (e.g. one specialist, one operator and one user).

All this has to fit into an overall integrated training schedule, which is a challenge of its own - remember that all astronauts and cosmonauts have individually tailored training plans.

Each crew member, while being a specialist for some systems, will be an operator or only a user for other systems. Consequently, the training program for each crew member is individually tailored to his or her set of tasks and pre-defined qualification levels.

Then the Mission Control Center (MCC) has to distribute the flight operations between the crew members and the MCC controllers.

Due to the fact that the potential of the ISS is limited, it is extremely important to maximize the efficiency of use, according to a budget constraint.

Nowadays, scheduling is performed manually without using any mathematical approach, based only on the experience of the employees. Besides, errors cumulate during the planning process and cause huge time and financial expenses. We hope that the considered approaches and models have the potential to reduce these expenses.

In this paper, the following problems are considered: The development of training plans for a crew of three cosmonauts, which is determined as the distribution of a pre-defined set of minimum qualification levels among the members of a crew, using the following criteria:

- minimizing the differences between the total time of the preparation of all crew members;
- minimizing the training expenses;
- maximizing the training level with a limited budget.

In Section 2, the cosmonaut training process is described. Mathematical models are given in Section 3, and approaches are considered in Section 4.

## 2 Description of the cosmonaut training

The sequence of the training program is based on four training phases:

1. General space training (GST) of candidates for cosmonauts;
2. Training in groups, separated by the type of manned spacecraft (MSC) or areas of specialization;
3. Training in approved crews for a specific space flight on MSC;

Passing the sequence of the stages of the training is mandatory for all Russian cosmonauts. The GST is performed for every candidate only once. The other stages can be performed repeatedly. Usually, the first three phases last 2,2 and 2.5 years, repspectively. In order to increase the efficiency and to decrease the expenses, the training time should be as short as possible.

### 2.1 General space training

The general space training provides the candidate cosmonauts with basic knowledge on space technology and science, basic medical skills and basic skills related to their future operational tasks, including those related to the station systems and operations.

The GST objectives are:

- to provide with knowledge and skills related to
- theoretical foundations of cosmonautics;
- principles of the design and the basis of the MSC, its service systems, scientific and special equipment;
- operation of the MSC, its serving systems, scientific and special equipment;
- theoretical foundations of scientific research and experiments carrying out at the MSC;
- systems of the manned orbital station;
- foreign MSC;
- the objects of the ground space infrastructure;
- interaction with the ground;
- working on a personal computer;
- conducting testing, research and experimentation on MSC;
- influence of dynamic factors of a space flight;
- working in space suits;
- Extra Vehicular Activity (EVA) in a hydrosphere and short-term weightlessness on flying laboratories;
- initial implementation of maintenance operations and maintenance (MOM), assembly and dismantling operations (ADO), handling operations (HO);
- scuba diving (see Fig. 3);
- landing under extreme conditions of various climatic zones (CZ)(see Fig. 4);
- flight and parachute;
- functions and responsibilities of crew members of the MSC;
- safety of space flight, including medical support;
- international and space law;
- English language, minimally required to prepare on the bases of the ISS program partners;
- to develop individual neuro-psychological resistance to adverse factors of a space flight and skills when working under difficult conditions of existence;
- to control and improve health;
- to identify individual psychophysiological characteristics of each candidate.

This training phase is a candidacy period and upon completion, successful candidates are certified as being career cosmonaut-test pilot or cosmonautresearcher. The GST has a duration of up to two years.

### 2.2 Training in groups, separated by the type of manned spacecraft or areas of specialization

The main purpose of this training phase is to study the MSC elements more in-depth. The cosmonauts learn to service and operate the different modules, systems and subsystems, and to fly and dock transport vehicles and an unmanned cargo carrier.

The objectives of this phase are to acquire a better knowledge and skills related to

- design, layout, on-board service systems, scientific and specialized equipment of a specific MSC;


Figure 3: Underwater low-gravity training

- work with on-board systems and scientific equipment;
- flight procedures and mission;
- typical operations on EVA in the Neutral Buoyancy Laboratory (NBL) and short-term weightlessness on flying laboratories and other technical facilities of the Cosmonaut Training (TFCT);
- active behavior in real stress;
- equipment inventory, MOM, ADO, HO;
- physical condition and functional capacity of the organism, high efficiency in the performance of professional tasks;
- on-board documentation;
- work with the Lead Operations Management Group of MCC;
- safety of the manned missions;
- operation and control of the MSC;
- typical accidents and emergency situations;
- English language (to a level that allows to participate in a training program at the facilities of the partner countries).

During this training phase, the cosmonauts ought to pass exams and tests in the relevant disciplines. The training in groups can be performed even after the formation of crews during the third training phase.

### 2.3 Training in crews

During this training phase, the cosmonauts learn everything they need to know for their mission. All crew members, prime and backup crews, selected for the space flight will train together.


Figure 4: Winter forest landing training
This is important not only because the crew members have to become known to each other (later they will spend about half a year together in the enclosed environment of the ISS), but they also learn to work efficiently together as a team and according to the distributed roles and responsibilities for which they are assigned to.

The crew tasks on the ISS are individually tailored, always considering the particular experience of the astronauts and the professional background.

The objectives of this phase are:

- to acquire knowledge and skills related to
- features and maintenance rules of the concrete MSC;
- program of the upcoming flight, on-board documentation and documents governing the rules of interaction between the crew members themselves and with the management teams and provide the flight crew with the code of conduct;
- control and operation of the MSC in regular modes and in case of emergencies;
- scientific research and experiments included into the flight program;
- functional duties in a crew;
- personal equipment (rescue suits, EVA suits, spacecraft chairs and others);
- concrete flight EVA tasks;
- rules and methods to ensure flight safety on a specific MSC;
- to form a psychological compatibility in the crew;
- to improve the interaction between the crew members, between the crews, and between the crews and the control groups;
- to ensure a good health, a high performance and the readiness to perform a biomedical section of the flight program;
- to conduct a pre-launch preparation with the crew;
- to increase the level of English up to the one needed to perform a space flight as a part of the international crew of the ISS.

There exist the following crew functions:

- on an MSC:
- commander,
- onboard engineer,
- onboard engineer-2,
- space flight participant.
- on an ISS:
- commander,
- onboard engineer,
- cosmonaut-researcher,
- space flight participant.

The training program contains a knowledge-based classroom training, as well as 'hands-on' using flight-like training mock-ups and simulators.

## 3 Mathematical models

The whole planning of the ISS cosmonaut training can be logically divided into two stages: the problem of volume planning and the problem of calendar planning.

The data for the volume planning problem is a set of onboard complexes and the required number of cosmonauts of different qualifications of each onboard complex. The aim is to distribute the training in qualifications of onboard complexes between the cosmonauts so that the total time of training becomes minimal.

The next important step of the planning process is the calendar planning. Once solved the volume problem of planning for each cosmonaut's defined set of onboard complexes for which it is necessary to be trained, it also raised the necessary qualifications for these onboard complexes. It is necessary to plan the training to minimize the time of preparation of the first crew, but to comply with the resource constraints and deadlines of the preparation of the other crews.

### 3.1 Volume planning problem

### 3.1.1 Model 1: Minimizing the differences.

## Notations

- $K$ - number of cosmonauts;
- $\mathcal{K}=\{1, \ldots, K\}$ - set of cosmonauts;
- $J$ - number of onboard complexes;
- $\mathcal{J}=\{1, \ldots, J\}$ - set of onboard complexes;
- $Q$ - number of qualifications;
- $\mathcal{Q}=\{1, \ldots, Q\}-$ set of qualifications;
- $\mathcal{O}_{j}$ - set of tasks on the onboard complex $j \in \mathcal{J}$;
- $\mathcal{O}_{j q}$ - set of tasks on the onboard complex $j \in \mathcal{J}$, available for a cosmonaut with qualification $q \in \mathcal{Q}$;
- $p_{j, q, e}, e \in\{0,1\}$ - amount of time needed to train an experienced ( $e=1$ ) or an inexperienced ( $e=0$ ) cosmonaut to qualification level $q \in \mathcal{Q}$ on the onboard complex $j \in \mathcal{J}$;
- $n_{j, q}$ - required number of cosmonauts with qualification level $q \in \mathcal{Q}$ on the onboard complex $j \in \mathcal{J}$;
- $D$ - maximum time of the training plan (constrained by the initial data);

Data A set of cosmonauts $\mathcal{K}$ should be trained. The cosmonauts could be experienced $(e=1)$ or inexperienced $(e=0)$. We have $J$ onboard complexes $j \in \mathcal{J}$. The number of qualifications can range from $Q=2$ to $Q=4$. We consider the case of the following $Q=3$ qualifications $q \in \mathcal{Q}$ : user $(q=1)$, operator $(q=2)$ and specialist $(q=3)$. It is assumed that all amounts $p_{j, q, e}$ and all numbers $n_{j, q}$ are known.
Besides we know that

$$
\mathcal{O}_{j_{1}} \bigcap \mathcal{O}_{j_{2}}=\varnothing, \quad j_{1} \neq j_{2}, \quad j_{1}, j_{2} \in \mathcal{J}
$$

## Variables

- $x_{k, j, q} \in\{0,1\}$ - Boolean variable: We have $x_{k j q}=1$ if cosmonaut $k \in \mathcal{K}$ should have the qualification level $q \in \mathcal{Q}$ on the onboard complex $j \in \mathcal{J}$;
- $\tau_{k}$ - total time of the training plan for cosmonaut $k \in \mathcal{K}$;

In our notation, the total training time of cosmonaut $k$ can be represented as the sum of the training times, assigned to the cosmonaut:

$$
\tau_{k}=\sum_{q \in \mathcal{Q}} \sum_{j \in \mathcal{J}} p_{j q e} x_{k j q} .
$$

## Objective function

$$
\begin{align*}
\left(\max _{k} \tau_{k}-\min _{k} \tau_{k}\right) & \rightarrow \min , \quad k \in \mathcal{K},  \tag{1}\\
\max _{k} \tau_{k} & \rightarrow \min , \quad k \in \mathcal{K},  \tag{2}\\
\min _{k} \tau_{k} & \rightarrow \max , \quad k \in \mathcal{K} . \tag{3}
\end{align*}
$$

## Constraints

$$
\begin{gather*}
\sum_{k \in \mathcal{K}} x_{k, j, q}=n_{j, q}, \quad j \in \mathcal{J}, q \in \mathcal{Q},  \tag{4}\\
\sum_{q \in \mathcal{Q}} x_{k, j, q} \leq 1, \quad j \in \mathcal{J}, k \in \mathcal{K}  \tag{5}\\
\sum_{q \in \mathcal{Q}} n_{j, q} \leq Q, \quad j \in \mathcal{J}  \tag{6}\\
\tau_{k} \leq D, \quad k \in \mathcal{K} \tag{7}
\end{gather*}
$$

In [10], it was shown that for this type of problem it is possible to use three different objective functions (1), (22), (3). Constraint (5) forbids that a cosmonaut has two different qualification levels on the same onboard complex. Constraint (4) requires that the number of cosmonauts, trained for each onboard complex, should be equal to the required number.

### 3.1.2 Model 2: Minimizing the expenses.

## Notations

- $K$ - number of cosmonauts;
- $\mathcal{K}=\{1, \ldots, K\}$ - set of cosmonauts;
- $J$ - number of onboard complexes;
- $\mathcal{J}=\{1, \ldots, J\}$ - set of onboard complexes;
- $Q$ - number of qualifications;
- $\mathcal{Q}=\{1, \ldots, Q\}-$ set of qualifications;
- $\mathcal{O}$ - set of all tasks provided on the ISS;
- $\mathcal{O}_{j}$ - set of tasks on the onboard complex $j \in \mathcal{J}$;
- $\mathcal{O}_{j q}$ - set of tasks on the onboard complex $j \in \mathcal{J}$, available for a cosmonaut with qualification $q \in \mathcal{Q}$;
- $p_{j, q, e}, e \in\{0,1\}$ - amount of time needed to train an experienced ( $e=1$ ) or an inexperienced $(e=0)$ cosmonaut to qualification level $q \in \mathcal{Q}$ on the onboard complex $j \in \mathcal{J}$;
- $n_{j, q}$ - required amount of cosmonauts with qualification level $q \in \mathcal{Q}$ on the onboard complex $j \in \mathcal{J}$;
- $D$ - maximum time of the training plan;
- $x_{k, j, q} \in\{0,1\}$ - Boolean variable: We have $x_{k j q}=1$ if cosmonaut $k \in \mathcal{K}$ should have the qualification level $q \in \mathcal{Q}$ on the onboard complex $j \in \mathcal{J}$;
- $\tau_{k}$ - total time of the training plan for cosmonaut $k \in \mathcal{K}$;
- $c_{k, j, q}$ - the cost of training cosmonaut $k \in \mathcal{K}$ to qualification level $q \in \mathcal{Q}$ on the onboard complex $j \in \mathcal{J}$;
- $W_{j}$ - required training level on the onboard complex $j \in \mathcal{J}$;
- $f_{q}$ - training level of a cosmonaut with the qualification level $q \in \mathcal{Q}$.


## Objective function

$$
\begin{equation*}
\sum_{k \in K} \sum_{j \in \mathcal{J}} \sum_{q \in \mathcal{Q}} c_{k, j, q} x_{k, j, q} \rightarrow \min . \tag{8}
\end{equation*}
$$

## Constraints

$$
\begin{equation*}
\sum_{q \in \mathcal{Q}} \sum_{k \in \mathcal{K}} f_{q} x_{k, j, q} \geq W_{j} \tag{9}
\end{equation*}
$$

(5):

$$
\sum_{q \in \mathcal{Q}} x_{k, j, q} \leq 1, \quad j \in \mathcal{J}, k \in \mathcal{K}
$$

(4):

$$
\sum_{k \in \mathcal{K}} x_{k, j, q}=n_{j, q}, \quad j \in \mathcal{J}, q \in \mathcal{Q},
$$

(7):

$$
\tau_{k} \leq D, \quad k \in \mathcal{K}
$$

### 3.1.3 Model 3: Maximizing the training level.

## Notations

- $K$ - number of cosmonauts;
- $\mathcal{K}=\{1, \ldots, K\}$ - set of cosmonauts;
- $J$ - number of onboard complexes;
- $\mathcal{J}=\{1, \ldots, J\}$ - set of onboard complexes;
- $Q$ - number of qualifications;
- $\mathcal{Q}=\{1, \ldots, Q\}$ - set of qualifications;
- $\mathcal{O}$ - set of all tasks provided on ISS;
- $\mathcal{O}_{j}$ - set of tasks on the onboard complex $j \in \mathcal{J}$;
- $\mathcal{O}_{j q}$ - set of tasks on the onboard complex $j \in \mathcal{J}$, available for a cosmonaut with qualification $q \in \mathcal{Q}$;
- $p_{j, q, e}, e \in\{0,1\}$ - amount of time needed to train an experienced ( $e=1$ ) or an inexperienced $(e=0)$ cosmonaut to qualification level $q \in \mathcal{Q}$ on the onboard complex $j \in \mathcal{J}$;
- $n_{j, q}$ - required number of cosmonauts with qualification level $q \in \mathcal{Q}$ on the onboard complex $j \in \mathcal{J}$;
- $D$ - maximum time of the training plan;
- $x_{k, j, q} \in\{0,1\}$ - Boolean variable: We have $x_{k j q}=1$ if cosmonaut $k \in \mathcal{K}$ should have the qualification level $q \in \mathcal{Q}$ on the onboard complex $j \in \mathcal{J}$;
- $\tau_{k}$ - total time of the training plan for cosmonaut $k \in \mathcal{K}$;
- $c_{k, j, l}$ - the cost of training cosmonaut $k$ to qualification level $q \in \mathcal{Q}$ on the onboard complex $j \in \mathcal{J}$;
- $W_{j}$ - required training level on the onboard complex $j \in \mathcal{J}$;
- $f_{q}$ - training level of a cosmonaut with the qualification level $q \in \mathcal{Q}$.
- $B$ - limit of the budget of a whole training process.


## Objective function

$$
\begin{equation*}
\sum_{k \in \mathcal{K}} \sum_{j \in J} \sum_{q \in \mathcal{Q}} f_{q} x_{k, j, q} \rightarrow \max . \tag{10}
\end{equation*}
$$

## Constraints

$$
\begin{equation*}
\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}} \sum_{q \in \mathcal{Q}} c_{k, j, q} x_{k, j, q} \leq B . \tag{11}
\end{equation*}
$$

(5):

$$
\sum_{q \in \mathcal{Q}} x_{k, j, q} \leq 1, \quad j \in \mathcal{J}, k \in \mathcal{K}
$$

(4):

$$
\sum_{k \in \mathcal{K}} x_{k, j, q}=n_{j, q}, \quad j \in \mathcal{J}, q \in \mathcal{Q}
$$

(7):

$$
\tau_{k} \leq D, \quad k \in \mathcal{K}
$$

### 3.1.4 Model 4: $K$-partition formulation.

Problem Consider now the special case when only one qualification level exists $(Q=1)$ and each onboard complex can be assigned only to one cosmonaut. Then

$$
\begin{aligned}
p_{e_{k}, j, q} & \rightarrow p_{e_{k}, j}, \\
n_{j, q} & \rightarrow n_{j}, \\
n_{j} & \leq K, \\
e=\{0,1\}, & k \in \mathcal{K}, j \in \mathcal{J}
\end{aligned}
$$

Let $\mathcal{J}$ be the set of onboard complexes and $\mathcal{J}_{k}$ be the subset of onboard complexes assigned to cosmonaut $k$ :

$$
\begin{gather*}
\bigcup_{k} \mathcal{J}_{k}=\mathcal{J}, \quad \mathcal{J}_{k}^{\prime} \cap \mathcal{J}_{k}=\emptyset  \tag{12}\\
\tau_{k} \quad \rightarrow \quad \tau_{k}=\sum_{j \in \mathcal{J}_{k}} p_{e_{k}, j}, \\
e=\{0,1\}, \quad k, k^{\prime} \in \mathcal{K}, k \neq k^{\prime}, j \in \mathcal{J} .
\end{gather*}
$$

The major goal is to find a partition of the set $\mathcal{J}$, which minimizes the difference between the total time of the preparation of all crew members.

Objective function Consider the same objective function as in the bottleneck assignment problem [10]:

$$
\begin{equation*}
\max _{k} \tau_{k} \rightarrow \min _{\mathcal{J}_{j}} \tag{13}
\end{equation*}
$$

In [2], algorithms for the "exact" solution of this problem were presented with a constraint on the number of jobs assigned to each cosmonaut. In [12], it was showen that these algorithms have a pseudo-polynomial time complexity. It was also proven that the problem is strongly $\mathcal{N} \mathcal{P}$-hard for general $m$. Thus, the algorithms in [2] cannot guarantee an optimal solution unless $\mathcal{P} \neq \mathcal{N} \mathcal{P}$, although they may be used as good heuristics [4].

From the statement of the problem, it can be seen that it is possible to use the objective function of one of the multi-way partition problems. In [10, it was shown that there were at least three of them: minimizing the largest subset sum (as in (133), maximizing the smallest subset sum, and minimizing the difference between the largest and smallest subset sums.

We will use the third objective function:

$$
\begin{equation*}
\delta=\left(\max _{k} \tau_{k}-\min _{k} \tau_{k}\right) \rightarrow \min _{\mathcal{J}_{j}} . \tag{14}
\end{equation*}
$$

### 3.2 Calendar planning problem

### 3.2.1 Model 5.

## Notations

- $\mathcal{C}=\{1, \ldots, C\}$ - set of crews, where the crews are sorted according to a non-decreasing order of their due dates;
- $\mathcal{K}_{c}=\left\{1, \ldots, K_{c}\right\}$ - set of cosmonauts in crew $c \in \mathcal{C}$;
- $\mathcal{K}=\bigcup_{c} \mathcal{K}_{c}$ - complete set of cosmonauts;
- $\mathcal{J}_{k}$ - set of tasks of cosmonaut $k$, which are required for the implementation of the training plan;
- $\mathcal{J}=\bigcup_{k} \mathcal{J}_{k}$ - set of all tasks;
- $\mathcal{T}=\{1, \ldots, T\}$ - set of the time moments (planning horizon);
- $p_{j}$ - execution time of the operation $j \in \mathcal{J}$;
- $\mathcal{R}=\{1, \ldots, R\}$ - set of resources;
- $r c_{j r}$ - amount of the resource $r$ needed to perform the task $j$;
- $r a_{r t}$ - amount of resource $r$ accessible during the time moment $t$;
- $D_{c}$ - due date of the crew $c \in \mathcal{C}$;
- $G=(J, \Gamma)$ - graph of the precedence relationships between the tasks: We have $\left(j, j^{\prime}\right) \in \Gamma$ if task $j$ must be performed before task $j^{\prime}$.
- $H=(J, \mathcal{H})$ - the graph of the strict precedence relationships between the tasks: We have $\left(j, j^{\prime}\right) \in \mathcal{H}$ if task $j^{\prime}$ must be performed immediately after the task $j$.


## Variables

- $x_{j t} \in\{0,1\}$ - Boolean variable: We have $x_{i j t}=1$ if and only if task $j$ starts the execution at time moment $t$;
- $S^{c} \in S$ - set of moments at which the execution of tasks from the set $J_{c}$ starts.


## Formulation of the problem

The optimization criterion is to minimize the total training time of the first crew:

$$
\begin{equation*}
C_{\max }\left(S^{1}\right) \rightarrow \min , \tag{15}
\end{equation*}
$$

where

$$
C_{\max }\left(S^{1}\right)=\max _{j \in \mathcal{I}^{1}}\left\{S_{j}+p_{j}\right\}
$$

Each task must be performed during the planning horizon:

$$
\begin{equation*}
\sum_{t=1}^{\mathcal{T}} x_{j t}=1, \quad j \in \mathcal{J} \tag{16}
\end{equation*}
$$

The resource limits must be respected:

$$
\begin{equation*}
\sum_{j \in \mathcal{J}} \sum_{t^{\prime}=t-p_{j}+1}^{t} r c_{j r} x_{j^{\prime}} \leq r a_{r t}, \quad \forall t \in \mathcal{T}, \quad \forall r \in \mathcal{R} \tag{17}
\end{equation*}
$$

The precedence relationships between the tasks must be respected:

$$
\begin{equation*}
\sum_{t^{\prime}=t-p_{j}+1}^{T} x_{j t^{\prime}}+\sum_{t^{\prime}=1}^{t} x_{j^{\prime} t^{\prime}} \leq 1, \quad \forall\left(j, j^{\prime}\right) \in \Gamma, \forall t \in \mathcal{T} . \tag{18}
\end{equation*}
$$

The strict precedence relationships between the tasks must be respected:

$$
\begin{equation*}
x_{j t}-x_{j^{\prime}\left(t+p_{j}\right)}=0, \quad \forall\left(j, j^{\prime}\right) \in \mathcal{H}, \forall t \in \mathcal{T} . \tag{19}
\end{equation*}
$$

The completion of the training time of the remaining crews may not exceed the due dates:

$$
\begin{equation*}
C_{\max }\left(S_{c}\right) \leq D_{c} \quad c=2, \ldots, C . \tag{20}
\end{equation*}
$$

## Formulation as an integer programming problem

The formulation below is a RCPSP. As it is known, such a problem can be represented as an integer programming problem.

We introduce the formal tasks:

- zero task $j_{0}, p_{0}=0$;
- final task for the first crew $j=J+1$ that should be performed after all tasks of the first crew, $p_{J+1}=0$;
- ... ;
- final task for the crew $c \in \mathcal{C}, j=J+c$ that should be performed after all tasks of the crew $c \in \mathcal{C}, p_{J+c}=0$;

Let $e s_{j}$ and $l s_{j}$ be the earliest and the latest moments at which task $j \in \mathcal{J}_{k}$ can be performed.

Then we get the following optimization problem:

$$
\begin{gather*}
\sum_{t=e s_{J+1}}^{l s_{J+1}} t x_{(J+1) t} \rightarrow \mathrm{~min} ;  \tag{21}\\
\sum_{t=e s_{j}}^{l s_{j}} x_{j t}=1 \quad j \in \mathcal{J} ;  \tag{22}\\
\sum_{t=e s_{j}}^{l s_{j}} t x_{j t}-\sum_{t=e s_{i}}^{l s_{i}} t x_{i t} \geq p_{i} \quad(i, j) \in G=(J, \Gamma) ;  \tag{23}\\
\sum_{t=e s_{j}}^{l s_{j}} t x_{j t}-\sum_{t=e s_{i}}^{l s_{i}} t x_{i t}=p_{i} \quad(i, j) \in H=(J, \mathcal{H}) ;  \tag{24}\\
\sum_{j \in \mathcal{J}} r c_{j r} \sum_{\tau=\sigma(t, j)}^{t} x_{j \tau} \leq r a_{r t} \quad t \in \mathcal{T}, r \in \mathcal{R},  \tag{25}\\
\text { with } \sigma(t, j)=\max \left(0, t-p_{j}+1\right) ; \\
\sum_{t=e s_{J+c}}^{l s_{J+c}} t x_{(J+c) t} \leq D_{c} \quad c=2, \ldots, C . \tag{26}
\end{gather*}
$$

where (21) is the price function that minimizes the total training time of the first crew, constraint (22) means that each task must be performed, constraints (23) and (24) describe the precedence and strict precedence relationships, respectively, constraint (25) is a resource constraint, and constraint (26) means that the completion of the training time of the remaining crews may not exceed the due dates.

Given the structure of the constraints as well as the size of the input data, one can observe that this problem is quite complex for modern solvers implementing standard algorithms of integer programming. Therefore, it is more likely, that an optimal solution cannot be obtained within a reasonable time. For this reason, we will develop heuristic algorithms for solving this problem.

### 3.2.2 Model 6.

Notations First, we introduce some time intervals:

- $W=\{1, \ldots,|W|\}$ - set of weeks in the planning period. The maximum is $|W|=130$ weeks ( 2.5 years). Because of the time previously given to specific operations, this set can be significantly reduced.


Figure 5: Time intervals

- $D_{w}=\{1,2,3,4,5,6,7\}-$ set of days per week, $w \in W$. If required, the operator can change this set, increasing or decreasing it (e.g., holidays, etc.).
- $H_{w d}=\{1, \ldots, 18,19\}$ - set of half-hour intervals of the day $d \in D_{w}$ of week $w \in W$.

It is assumed that the first interval begins at 9.00 a.m. and ends latest at $6.00 \mathrm{p} . \mathrm{m}$., followed by the dinner. Due to the dinner, it is necessary to divide the days in the model. In some cases, according to the information available to us, a typical schedule may increase the duration of the day for 1 hour (two intervals). Probably, the following process is possible: a feasible schedule with the current set of intervals cannot be developed, the program indicates where the conflict is, the operator decides to extend the working day, and to re-develop the schedule. As in a typical schedule there are very few operations that take no more than half an hour, perhaps the partition of the working day by hours (i.e., not by half of an hour) can be done. Then we have $H_{w d}=\{1, \ldots, 9\}$ and a significantly smaller dimension.

It will be convenient to work with restrictions such as "not more than 2 times a week", "in the morning", etc. On the other hand, for the calculation of the duration of the steps, it is necessary to have a linear decomposition of the planning horizon. To do this, let us arrange all triples $(w, d, h)$ in lexicographical order and to each triple, we associate its number: $(w, d, h) \rightarrow$ $t(w, d, h)$, where $t \in T=\{1, \ldots,|T|\},|T|$ is the number of triples (see Fig. 5).


Figure 6: Training schedule for all crews.

We denote the set of all triples $(w, d, h)$ as $Y$ :

$$
Y=\left\{(w, d, h) \mid w \in W, d \in D_{w}, h \in H_{w d}\right\}
$$

The crews start the training at different moments (see. Fig. 6). Therefore, over the period of 2.5 years, some cosmonauts have already mastered a part of the operations and thus, each cosmonaut has its own set of current operations.

Denote by $Y(k, j)$ the set of all possible time intervals for performing task $j$ by cosmonaut $k$. In this case, we do not consider the days when cosmonaut $k$ is on vacation and consider time constraints (limits $e_{j}$ and $l_{j}$ ).

Next, we introduce the basic notations.

- $C=\{1, \ldots,|C|\}-$ set of crews.
- $K_{c}-$ set of cosmonauts in the crew $c \in C$. Usually, $K_{c}=\{1,2,3\}$.
- $K$ - the complete set of cosmonauts.
- $J_{c}-$ set of tasks of the crew $c \in C$.
- $J_{k}$ - set of tasks of cosmonaut $k$, which are required for the implementation of the training plan. We divide this set into the following subsets:
- $J_{k}^{T}$ - set of technical tasks of the cosmonaut $k$, all tasks with onboard complexes are contained in it. Denote all tasks for the onboard complexes as $J_{k}^{B}$.
- $J_{k}^{F}$ - set of physical training tasks of the cosmonaut $k$ (which last 2 hours, i.e., we have 4 intervals).
- $J_{k}^{A}$ - set of administrative tasks of the cosmonaut $k$.
- $J_{k}^{L}$ - set of language lessons of the cosmonaut $k$ (which last 2 hours, i.e., we have 4 intervals).

We distinguish subsets in the set $J_{k}^{B}$ which contain the tasks of the onboard complexes $J_{k}^{B_{1}}, J_{k}^{B_{2}}, \ldots, J_{k}^{B_{m_{k}}}$, where $m_{k}$ denotes the number of onboard complexes which should examine cosmonaut $k$.

- $p_{j}-j \in J$ execution time.
- $R=\{1, \ldots,|R|\}$ - set of resources. Each cosmonaut is also a resource that is available in amount 1 at any time.
- $r c_{j r}$ - amount of the resource $r$ needed to perform the task $j$.
- $r a_{r w d h}$ - amount of the resource $r$ accessible during time interval $h$ of a day $d$, week $w$.
- $e_{j}, l_{j}$ - the earliest and the latest moments at which task $j \in J$ can be performed.
- $J_{k}^{\text {bound }}$ - set of tasks for which time constraints are defined. Due dates can also be described using these boundaries.
- $J_{k}^{d a y}$ - set of tasks that should be performed during one day.
- $J_{k}^{w e e k}$ - set of tasks that should be performed during one week.
- $J_{c}^{123}$ - set of tasks that should be performed by all cosmonauts of the crew $c \in C$. Similarly, we define the sets $J_{c}^{12}, J_{c}^{13}, J_{c}^{23}$.
- $G=(J, \Gamma)$ - the graph of the precedence relations between the tasks: We have $\left(j, j^{\prime}\right) \in \Gamma$ if task $j$ must be performed before the task $j^{\prime}$.
- $H=(J, \mathcal{H})$ - the graph of the strict precedence relations between the tasks: We have $\left(j, j^{\prime}\right) \in \mathcal{H}$ if task $j^{\prime}$ must be performed immediately after the task $j$.

We can divide the operations that take more than one day into one-day operations. For these operations, we can introduce the graph of the "almost strict" precedence relations $S H$. If $\left(j_{1}, j_{2}\right) \in S H$, then operation $j_{2}$ should be performed after operation $j_{1}$ and there should be one time interval between them. So, we get a sequence of one-day operations divided by the dinner instead of a multi-day operation.

## Variables

- $x_{k j w d h}$ - Boolean variable: We have $x_{k j w d h}=1$ if and only if the cosmonaut $k$ starts the task $j$ from the interval $h$ of the day $d$ of week $w$;
- $y_{i k w}$ - Boolean variable: We have $y_{i k w}=1$ if and only if the cosmonaut $k$ trains for the onboard complex $i$ during the week $w$.

Constraints The following relations between the variables have to be satisfied:

$$
\begin{gather*}
\sum_{j \in J_{k}^{B_{i}}} x_{k j w d h} \leq y_{i k w},  \tag{27}\\
\forall k \in K, \forall i \in\left\{1, \ldots, m_{k}\right\}, \forall(w, d, h) \in Y .
\end{gather*}
$$

The resource limits have to be respected:

$$
\sum_{k \in K} \sum_{j \in J} r c_{j r} \sum_{\substack{\left(w^{\prime}, d^{\prime}, h^{\prime}\right) \in Y \\ t(w, d, h)-p_{j}^{\prime}+1 \leq t\left(w^{\prime}, d^{\prime}, h^{\prime}\right) \leq t(w, d, h)}} x_{k j w^{\prime} d^{\prime} h^{\prime}} \leq r a_{r w d h},
$$

In this inequality, for each $(w, d, h) \in Y$, we consider only the operations that are performed at this interval, i.e., which started in the interval $[t(w, d, h)-$ $\left.p_{j}+1, t(w, d, h)\right]$.

Each cosmonaut should perform all required tasks:

$$
\begin{equation*}
\sum_{(w, d, h) \in Y(k, j)} x_{k j w d h}=1, \quad \forall k \in K, \forall j \in J_{i} . \tag{29}
\end{equation*}
$$

Each cosmonaut must have 4 hours ( 2 tasks for 2 hours) of physical training per week:

$$
\begin{equation*}
\sum_{j \in J_{k}^{F}} \sum_{d \in D_{w}} \sum_{h \in H_{w d}} x_{k j w d h} \leq 2, \quad \forall k \in K, \forall w \in W \tag{30}
\end{equation*}
$$

Similarly, we can set constraints on the language study. Each cosmonaut must have 4 hours of language lessons per week at the beginning of the whole training:

$$
\begin{equation*}
\sum_{j \in J_{k}^{L}} \sum_{d \in D_{w}} \sum_{h \in H_{w d}} x_{k j w d h} \leq 2, \quad \forall k \in K, \forall w \in W \tag{31}
\end{equation*}
$$

Similarly, we have constraints for the administrative tasks:

$$
\begin{equation*}
\sum_{j \in J_{k}^{A}} \sum_{d \in D_{w}} \sum_{h \in H_{w d}} x_{k j w d h} \leq 4, \quad \forall k \in K, \forall w \in W \tag{32}
\end{equation*}
$$

It is forbidden to plan more than 4 hours of training for one onboard complex per day:

$$
\begin{gather*}
\sum_{j \in J_{k}^{B_{i}}} \sum_{h \in H_{w d}} p_{j} x_{k j w d h} \leq 8,  \tag{33}\\
\forall k \in K, \forall i \in\left\{1, \ldots, m_{k}\right\}, \forall w \in W, \forall d \in D_{w} .
\end{gather*}
$$

It is forbidden to plan the training for more than two onboard complexes per week:

$$
\begin{equation*}
\sum_{k=1}^{m_{k}} y_{i k w} \leq 2, \quad \forall k \in K, \forall w \in W \tag{34}
\end{equation*}
$$

There are time limits for some tasks:

$$
\begin{gather*}
x_{k j w h d}=0,  \tag{35}\\
\forall k \in K, \forall j \in J_{k}^{\text {bound }}, \forall(w, d, h) \in Y: t(w, d, h) \leq e_{j}-1, \\
x_{k j w h d}=0,  \tag{36}\\
\forall k \in K, \forall j \in J_{k}^{\text {bound }}, \forall(w, d, h) \in Y: t(w, d, h) \geq l_{j}+1 .
\end{gather*}
$$

The precedence relations (if $\left(j_{1}, j_{2}\right) \in \Gamma$, then task $j_{1}$ must be performed before task $j_{2}$ ) must be respected:

$$
\begin{gather*}
\sum_{(w, d, h) \in Y\left(k, j_{2}\right)} t(w, d, h) x_{k j_{2} w d h}-\sum_{(w, d, h) \in Y\left(k, j_{1}\right)} t(w, d, h) x_{k j_{1} w d h} \leq p_{j_{1}},  \tag{37}\\
\forall k \in K, \forall\left(j_{1}, j_{2}\right) \in \Gamma .
\end{gather*}
$$

The strict precedence relations must be respected:

$$
\begin{gather*}
\sum_{(w, d, h) \in Y\left(k, j_{2}\right)} t(w, d, h) x_{k j_{2} w d h}-\sum_{(w, d, h) \in Y\left(k, j_{1}\right)} t(w, d, h) x_{k j_{1} w d h}=p_{j_{1}},  \tag{38}\\
\forall k \in K, \forall\left(j_{1}, j_{2}\right) \in \mathcal{H} .
\end{gather*}
$$

We must consider that $H \subseteq G$.
The "almost strict" precedence relations must be respected:

$$
\begin{gathered}
\sum_{(w, d, h) \in Y\left(k, j_{2}\right)} t(w, d, h) x_{k j_{2} w d h}-\sum_{(w, d, h) \in Y\left(k, j_{1}\right)} t(w, d, h) x_{k j_{1} w d h}=p_{j_{1}}+1, \\
\forall k \in K, \forall\left(j_{1}, j_{2}\right) \in \mathcal{S H} .
\end{gathered}
$$

Remark 3.1. If the date of the vacation of the cosmonaut is not defined and must be determined during the planning phase, it can be considered as an additional task of appropriate length.

Remark 3.2. The lunch time can be strictly fixed or it can be considered as a task with time constraints (for example, from 12.00 a.m. to 3.00 p.m.).

Objective function Since the first crew starts before the others, it has the priority in the planning phase, and a possible formulation of the problem includes the minimization of its total training time. To do this, we introduce an additional variable $t^{f}$ and constraints on the additional last task $t_{k}^{f}$ for each cosmonaut $k \in K_{1}$ :

$$
\begin{equation*}
t(w, d, h) x_{k j_{k}^{f} w d h} \leq t^{f}, \forall k \in K_{1}, \forall(w, d, h) \in Y \tag{40}
\end{equation*}
$$

In this case, the objective function is:

$$
\begin{equation*}
\min t^{f} \tag{41}
\end{equation*}
$$

## 4 Approaches

### 4.1 Algorithm-3.0

Consider the model 1.
We combine the variables $x_{k, j, q}$ of the same qualification level and onboard complex into a vector $\vec{x}_{j, q}=\left\{x_{1, j, q}, x_{2, j, q}, x_{3, j, q}\right\}$.

Step 1 First, we are going to find all onboard complexes which require all three cosmonauts or none of them have the same qualification level. Then there exists only one option of the training plan satisfying this condition:

$$
\begin{array}{ll}
\vec{x}_{j q}=\{1,1,1\}, & \forall j, q: n_{j, q}=3, \\
\vec{x}_{j q}=\{0,0,0\}, & \forall j, q: n_{j, q}=0 .
\end{array}
$$

Step 2 Let $J^{\prime}$ be the number of onboard complexes left after the previous step. Consider this problem as $J^{\prime}$ independent subproblems. For each subproblem, we will find the minimum separately.

It can be interpreted as $J^{\prime}$ boxes (Fig. 7) with

$$
C_{j}=C_{3}^{n_{j, S}} \cdot C_{3-n_{j, S}}^{n_{j, O}} \cdot C_{3-n_{j, S}-n_{j, O}}^{n_{j, O}}, \quad j=\overline{1, J^{\prime}}
$$



Figure 7: Each box is an onboard complex, and each item is an option for the distribution.
objects in each of them. It is allowed to take only one of them from each box.

Considering all subproblems as independent ones reduces the number of variations from

$$
\prod_{j=1}^{J^{\prime}} C_{j} \quad \leq 6^{J^{\prime}}
$$

to

$$
\sum_{j=1}^{J^{\prime}} C_{j} \quad \leq 6 J^{\prime}
$$

Step 3 For each required qualification level on each onboard complex, a cosmonaut should be determined. Due to this fact, the problem is less complicated than that where a subset of qualification levels should be chosen on which the best value is distinguished like in a knapsack problem.

At this step, we sort the values $p_{1, j, q}$ in non-increasing order to cover the difference among the total time of the training plans of each cosmonaut at the next steps of the algorithm.

Step 4 Perform $J^{\prime}$ iterations.
At iteration $j$, find $\vec{x}_{j, S}, \vec{x}_{j, O}, \vec{x}_{j, U}$ such that

$$
\min _{\vec{x}_{j, S},}^{\vec{x}_{j, O}, \vec{x}_{j, U}}\left(R_{j}\left(\vec{x}_{j, S}, \vec{x}_{j, O}, \vec{x}_{j, U}\right)\right)=\min _{\vec{x}_{j}, S}, \vec{x}_{j, O}, \vec{x}_{j, U}\left(\sum_{k^{\prime}=1}^{3} \sum_{k^{\prime}>k}\left|\tau_{k, j}-\tau_{k^{\prime}, j}\right|\right),
$$

$$
\begin{aligned}
& \tau_{k, 0}=0, \\
& \tau_{k, j}=\tau_{k, j-1}+\sum_{q=S, O, U} c_{e_{k}, j, q} x_{k, j, q},
\end{aligned}
$$

$k=\overline{1,3}, j=\overline{1, J^{\prime}}, e_{k}=1$ if and only if cosmonaut $k$ is experienced and $x_{k, j, q}$ satisfies the constraints (4) and (5).

If $n_{j, q}=1$, then

$$
\vec{x}_{j, q}=\{1,0,0\} \quad \text { or } \quad\{0,1,0\} \quad \text { or } \quad\{0,0,1\} .
$$

If $n_{j, q}=2$, then

$$
\vec{x}_{j, q}=\{1,1,0\} \quad \text { or } \quad\{0,1,1\} \quad \text { or } \quad\{1,0,1\} .
$$

Complexity At each iteration, we have to calculate the R maximum of

$$
C_{j}=C_{3}^{n_{j, S}} \cdot C_{3-n_{j, S}}^{n_{j, O}} \cdot C_{3-n_{j, S}-n_{j, O}}^{n_{j, O}}=6
$$

times with $n_{j, S}=n_{j, O}=n_{j, S}=1$.
Due to this fact, if there are $n$ onboard complexes, the number of operations is equal to $O(n)$.

### 4.2 Partition algorithm

Consider the model 4.

### 4.2.1 $\quad \mathcal{N} \mathcal{P}$-completeness

The first goal in the analysis is an $\mathcal{N} \mathcal{P}$-completeness proof for the problem with the criterion (14) subject to (12). Obviously, the problem is $\mathcal{N} \mathcal{P}$-hard. By a local replacement [5], it is possible to show that the problem is also $\mathcal{N} \mathcal{P}$-complete. Suppose that all cosmonauts have an equal training time for every job. Then the problem reduces to the multiway-partition problem which is indeed $\mathcal{N} \mathcal{P}$-complete. So, the cosmonaut assignment problem is $\mathcal{N} \mathcal{P}$-complete as well.

### 4.2.2 Algorithms

Heuristic algorithm As a first approximation, a greedy algorithm based on heuristic considerations can be used. At each step, the algorithm fixes a job and makes an assignment to the cosmonaut, which will have the minimum objective function value (14). The complexity of such an algorithm
is $O\left(m^{2} n\right)$, where $m$ is the number of cosmonauts and $n$ is the number of jobs.
Example (Tables 1-4)

Table 1: Initialization

| Job $\rightarrow$ | First | Second | Third |
| :---: | :---: | :---: | :---: |
| Commander | 1 | 2 | 3 |
| Engineer | 2 | 4 | 6 |
| User | 2 | 1 | 2 |

Table 2: Assignment of the first job

| Job $\rightarrow$ | First | Second | Third |
| :---: | :---: | :---: | :---: |
| Commander | 1 | 2 | 3 |
| Engineer | 2 | 4 | 6 |
| User | 2 | 1 | 2 |

Table 3: Assignment of the second job

| Job $\rightarrow$ | First | Second | Third |
| :---: | :---: | :---: | :---: |
| Commander | 1 | 2 | 3 |
| Engineer | 2 | 4 | 6 |
| User | 2 | 1 | 2 |

Table 4: Assignment of the third job

| Job $\rightarrow$ | First | Second | Third |
| :---: | :---: | :---: | :---: |
| Commander | 1 | 2 | 3 |
| Engineer | 2 | 4 | 6 |
| User | 2 | 1 | 2 |

Here we have $\delta=3$, while the optimal objective function value is 0 .
Observation It is possible to find an instance which will have an arbitrary error as it is shown in Table 5.

Table 5: "Bad" instance

| $[\mathrm{h}] \mathrm{Job} \rightarrow$ | First | Second | Third |
| :---: | :---: | :---: | :---: |
| Commander | 0 | A | A |
| Engineer | A | A | A |
| User | A | A | A |

Here A is a random number. So the error of the heuristic algorithm will be equal to A , but the optimal objective function value is equal to 0 .

According to the proposed algorithm, a program was written and tests were carried out on real data provided by the Yu. A. Gagarin research \& test cosmonaut training center. In spite of the above observation, the error of the greedy algorithm does not exceed $10 \%$ of the optimum value.

Exact algorithm Assume that we have found an optimal solution. Then any permutation of the work of one cosmonaut would lead to the fact that the objective function will increase or remain the same. If there is a permutation, leading to a decrease in the objective function, it can be argued that the chosen solution is not optimal. Let a solution be obtained, for example by using the heuristic algorithm described earlier. Then we can find out whether it is optimal using the following lemma.

Lemma 4.1. Let $\mathcal{J}_{k}^{\prime}=\left\{\mathcal{J}_{1}^{\prime}, \ldots, \mathcal{J}_{m}^{\prime}\right\}$ be the feasible subsets of the set $\mathcal{J}$. Moreover, let all cosmonauts be sorted in non-increasing order of the keys $y_{k}$. Then it is possible to check whether an optimal solution is obtained with $O\left(\left|\mathcal{J}_{1}^{\prime}\right|\left(m+\sum_{2 \leq s \leq m}\left|\mathcal{J}_{s}^{\prime}\right|\right)\right)$ operations in the worst case.

Proof An exhaustive search can be avoided if we consider that the objective function can only increase or remain the same, if the cosmonauts with maximum or minimum time of the training are not involved into the permutation.

## Algorithm

1. Find a solution by the heuristic algorithm.
2. Sort the cosmonauts in non-increasing order of the keys $y_{k}$.
3. Check whether the solution is optimal using Lemma 4.1.
4. If the solution is not optimal, apply a permutation that leads to a decrease of $\delta$.
5. Repeat step 2.

### 4.3 Integer programming

The software package CPLEX has been used for the solution of the problem.

1. The integer constraints were relaxed, and then the linear programming problem was solved.
2. The original problem was solved by the branch-and-bound method using the solution of the relaxed problem as a lower bound.
3. Constraints on the one of the variables were added, and then the algorithm iterates again.

## 5 Comparison of the algorithms

The two proposed algorithms were tested on real data and compared with an integer programming technique. All results are shown in Table 6. The running time of the exact branch-and-bound method, implemented on CPLEX, was limited to 15 minutes.

Table 6: Numerical Experiments

| Sample | Experience | Alg.-3.0 |  |  | Partition alg. |  |  | Integer Progr.(CPLEX) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | max | min | $\delta$ | max | min | $\delta$ | max | min | $\delta$ |
| $\begin{gathered} \min \\ \text { SQRM } \end{gathered}$ | 3 Inexperienced | 883.25 | 881.00 | 2.25 | 887.8 | 886.75 | 1.05 | 888.05 | 887.75 | 0.3 |
|  | 3 Experienced | 570.00 | 568.50 | 1.5 | 570.5 | 569 | 1.5 | 570 | 569.5 | 0.5 |
|  | 1 Exp 2 Inexp |  |  |  |  |  |  | 697.25 | 695.25 | 2 |
|  | 2 Exp 1 Inexp |  |  |  |  |  |  | 616.5 | 612.75 | 3.75 |
| 6 months | 3 Inexperienced |  |  |  | 266.25 | 265 | 1.25 | 265.75 | 265.2 | 0.55 |
|  | 3 Experienced |  |  |  | 234.2 | 233 | 1.2 | 233.75 | 233.25 | 0.5 |
|  | 1 Exp 2 Inexp |  |  |  |  |  |  | 244.45 | 244 | 0.45 |
|  | 2 Exp 1 Inexp |  |  |  |  |  |  | 233.75 | 233.25 | 0.5 |
| $2$ <br> years | 3 Inexperienced |  |  |  | 661.25 | 657.5 | 3.75 | 659.85 | 659.75 | 0.1 |
|  | 3 Experienced |  |  |  | 353.5 | 353.05 | 0.45 | 353.5 | 353 | 0.5 |
|  | 1 Exp 2 Inexp |  |  |  |  |  |  | 484.05 | 481.75 | 2.3 |
|  | 2 Exp 1 Inexp |  |  |  |  |  |  | 393.5 | 382.5 | 1 |
| STFL | 3 Inexperienced |  |  |  | 925.75 | 922.25 | 3.5 | 925 | 924.8 | 0.2 |
|  | 3 Experienced |  |  |  | 587 | 586.5 | 0.5 | 587 | 586.5 | 0.5 |
|  | 1 Exp 2 Inexp |  |  |  |  |  |  | 731.5 | 730.75 | 0.75 |
|  | 2 Exp 1 Inexp |  |  |  |  |  |  | 628.75 | 628 | 0.75 |

It can be observed that the third algorithm based on integer programming techniques has the best accuracy, but it still cannot be guaranteed that the values obtained are indeed optimal. For example, it can be seen when compared with the second algorithm for the data " 2 years" for three experienced cosmonauts. In this case, the greedy algorithm gives a better solution. However, among all data, this is the only case.

The essential difference between the first and second algorithms on one side and the third algorithm on the other side is that two qualification levels with the highest training time would be assigned to two different cosmonauts, whereas this is not mandatory in the third algorithm. It can be found that obvious correlations in the distribution of the work with medium and small durations in the second and third algorithms cannot be observed.

From our experiments, it can be seen that the solution quality of the first two algorithms is similar.

## 6 Conclusion

This article described the process of the training of cosmonauts and the relevance of the planning phase. Some models and methods for solving the training scheduling problem were suggested. For the problem of volume planning, several models and three algorithms were presented. To solve the calendar problem, integer programming methods problems and RCPSP algorithms will be used. In the future, we plan to develop an automated workplace (AWP), which allows to automate the process of planning the training of the ISS crews.

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