An Integrated Approach for Maintenance and Delivery Scheduling in Military Supply Chains

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Abstract
This paper focuses on an integrated demand-responsive scheduling of operations within a two-echelon military supply network consisting of maintenance and transportation operations during peacetime. The paper emphasizes on the continuous operation of military's training facilities. Integration of operations is carried out by embedding an additional intermediary (command) module into the maintenance-transportation model. We analyze two performance measures: time of response (defining the system's ability to react speedily to military requirements) and military effectiveness (which defines the ability of the military supply network to deliver the right product at the right time). Using integer programming techniques combined with a heuristic algorithm, we derive a new method for coordinating maintenance and transportation operations in military supply networks. The standard simulation tool ARENA 11.0 is used to implement the integrated scheduling/planning method. The results of a computational experiment are presented.

Keywords: Integrated scheduling; Heuristics; Simulation; Routing; Military supply network.

MSC classification: 90 B 35, 90 C 11, 90 C 27

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1. Introduction

The primary goal of large-scale commercial and military supply networks is to procure, produce, repair and distribute requested civil or military commodities, items and other goods and services in a timely and cost-effective way. This paper focuses on coordinating maintenance and transportation operations in military supply networks consisting of suppliers, maintenance units, warehouses, distribution centers, retailers and military bases (National Materials Advisory Board 2008).

Since an effective logistic support of military operations entails the rapid supply of the right product to the right location at the right time, lack of coordination among different components that form the military supply network can impair the effectiveness and responsiveness of military forces in the theater of operations. There are numerous examples from military history that illustrate this fact. For example, during the Gulf War (1990-1991) lack of a coordinated supply network prevented logisticians in US Marines’ 1st Battalion from transferring critical tank parts to the army in time (Kirkland 2000). The key issue in terms of improved logistics is having the spare part, where it is needed. For example, during the operation Desert Storm (1991) $2.7 billion worth of spare parts went unused according to a 1992 report from the US General Accounting Office. It is estimated that, if the US Army had an effective cargo-tracking, it would have saved about $2 billion.

In this paper, we focus on integrated, demand-responsive scheduling of operations within military supply networks in support of a continuous operation of training facilities.

In our study we follow the guidelines set out in the military standard MIL-STD 1378D (1990). This standard provides detailed procedures for coordinating all training efforts and tasks in military training programs. It also identifies the general requirements of maintenance and transportation modules within the various training programs. In accordance with the recommendations of the standard, we select and analyze a two-echelon military supply network that includes maintenance (repair) and transportation operations. Maintenance involves restoring operational capabilities by maintaining and/or replacing damaged parts. A lack of integration between these two operations can cause a "domino effect". For example, a one-hour delay in supplying a necessary spare part may result in a one-day delay in transporting the repaired parts to the end-customers (combat units or combat support units); if the military training is based on a week-long cycle, the transportation delay will hold up the next training cycle by a week.

Two performance measures: time of response and military effectiveness are introduced and then mathematically described in order to analyze the effectiveness of the military supply network that we define. To obtain an optimal solution of the proposed integrated planning-scheduling problem, we employ a new heuristic technique based on a two-stage search algorithm combined with integer programming techniques. The implementation of this heuristic method is based on the discrete event simulation tool Arena 11.0.
The paper is organized as follows. Section 2 overviews the related work. Section 3 presents the description of demand-responsive military supply networks. Sections 4 and 5 introduce new integrated scheduling models. A heuristic solution technique is given in Section 6. The results of the computational experiments are presented in Section 7. Section 8 concludes the paper.

2. Literature review

This section focuses on three main categories: (1) mathematical models in military supply networks; (2) the role of performance measures in supply chain management; and (3) analysis of models for integrated production and transportation operations.

2.1. Mathematical models in military supply networks

Kress (2002) investigates the management of warfare logistics in planning, implementing and controlling military operations. Yoho et al. (2013) discuss how defense logistics has been studied in the past and how it is being studied today. Proposing a research agenda for future work, they suggest that the most important direction for military logistics in the future is to improve coordination of the relationships between different actors in the supply system and maintenance and the overhaul of both complex and aging weapon systems. Akgün and Tansel (2007) present the deployment planning problem (DPP) as a problem in preparing the physical movement of military units, positioned at geographically dispersed locations, from their military bases to their deployment destinations, subject to various scheduling and routing constraints and different types of transportation assets. To solve the DPP, a mixed integer program model is formulated.

Focusing on the design of an efficient military supply network for recovering failed combat vehicles in unpredictable military environments, Barahona et al. (2007) use a new integrated approach, called chain-centric logistics (NCL). This approach is based on controlling physical inventory and distribution processes across the whole supply network. Two mathematical optimization models for the inventory and distribution modules are formulated and solved by using integer programming algorithms and simulation. A shortfall of the NCL model is that the maintenance process is not considered.

McGee et al. (2005) use simulation to describe the operation of a multi-level inventory system for an air force in order to analyze how effectively transportation can reduce costs and delay times and improve overall readiness in the logistics network. The authors only focus on finding an optimal transportation utilization of the multi-echelon inventory system; the optimization and coordination of the maintenance operation within the transportation module are not taken into account. Wilhite et al. (2014) consider a problem related to the structure of the military repair supply network in the USA Army. The authors explore when it is in the best interest of the army to develop internal capabilities (repair manufactures), rather than to use external ones. Overholts et al. (2009) focus on developing a missile maintenance and security forces scheduling model for the US Air Force. Since maintaining the
inter-continental ballistic missile weapon system requires that maintenance and security teams travel to some 150 launch sites, a two-stage model that maximally covers the locations was developed. In order to investigate the military replenishment system of the Chinese Army, Chang et al. (2007) develop a simulation model based on a system dynamic approach. Recently, Fan et al. (2010) use this approach to study the bullwhip effect in a military maintenance supply system within the Chinese Army’s repair and logistic system. Joo (2009) develops a dynamic approach for scheduling preventive maintenance at a depot with limited availability of spare modules and other constraints. A backward allocation algorithm is proposed and applied to schedule the preventive maintenance of an engine module installed in advanced jet trainers.

2.2. The role of performance measures in supply chain management

The difficulty in developing appropriate performance measures depends on the complexity of the supply network structure and the number of echelons and the facilities in each echelon. Beamon (1999) provides an overview and evaluation of the performance measures used in supply chain (SC) models. The author indicates that the SC performance measurement system must contain at least one of three types of performance measures identified as necessary components in any SC performance measurement system: resources, outputs, and flexibility. Measuring the performance of the military chains is also important for recognizing troubled sectors, defining success and assessing operational capabilities. Burns et al. (2010) argue that the primary objective of the military supply network (SC) is to attain a specific state of readiness at the lowest possible overall cost. They claim that the metric for military SC success is readiness for war and not the presumed profitability of the SC.

The combination of streamlined procurement, supply, distribution, and lean practices is fundamental to create an efficient military supply chain management system that will reduce the minimal customer wait time (see Leiphart 2001). The effectiveness of military supply networks can also be analyzed using indicators such as communication volume, logistics volume, and waiting times (see Gibson 2004). Once hindrances are identified, logisticians can implement the right procedures to unblock the flow of material and increase the supply chain velocity. In order to develop an efficient military supply network for spare parts, the RAND Corporation (see Johnson and Levite 2003) introduced for the US Army a new methodology, called define-measure-improve (DMI) that consists of two major indicators, namely, customer wait-time (CWT) and requisition wait-time (RWT). The CWT measures the supply network performance from the unit perspective: the time it takes to satisfy a request for a part necessary for making a repair. In contrast, the RWT assesses how well the supply network for spare parts serves itself, i.e., how much time it takes to meet a unit’s demand. Among other works in this field, we would like to mention the paper by Wang (2006). In the works of Leiphart (2001) and Gibson (2004), the authors omit a comprehensive definition of military SC flexibility as well as performance measures to evaluate it, where SC flexibility is defined as the ability of the SC to respond to the changing requirements of purchased components in terms of volume. Sokri
addresses these gaps and seeks to develop performance measures to assess the flexibility of a military SC. In his study, the author identifies two major groups of flexibility measures: volume and delivery.

2.3. Analysis of models for integrated production and transportation operations

As a way to improve performance measures and to render the whole supply network more effective, many researchers in the field consider the production-inventory-distribution process as an integrated system, where inventory is an integral part of a production and distribution system. Dhaenens-Flipo and Finke (2001) introduce a mixed integer programming model based on multi-products, facilities and periods. Their interest is to simultaneously obtain the optimal quantity of manufactured products on a specific production line and the optimal quantity of products transported from the stock to the end-customers. Low and Chang (2013) investigate a two-echelon supply network in which the orders first have been processed by a distribution center and then delivered to the retailers with respect to the prescribed time windows. A nonlinear mathematical model is formulated and solved by employing a genetic algorithm. The work by Steinrücke (2011) focuses on determining an optimal scheduling for integrated production-transportation operations within an aluminium supply network to minimize total cost. A monolithic mathematical model is formulated to tackle the problem. The obtained results show that employing the monolithic model in determining optimal solutions for practical instances is not a viable procedure. Therefore, the relax-and-fix heuristics (RF heuristics) are developed to obtain best possible solutions in reasonable computational time. There are several other studies that focus on special types of integrated production and distribution scheduling problems, where manufacturers produce their product only when the customer places the order, that is, a stock of finished commodities is forbidden (a so-called make-to-order approach) (see, for example, Chen and Vairaktarakis 2005). The scheduling of the internal integrated production-transportation operations in manufactures is presented in the work of Levner et al. (2009). The authors develop a so-called hierarchically based heuristic approach (consisting of batching and local search heuristics, where they suggest to use adaptations of the heuristics suggested in Sotskov et al. (1996), Danneberg et al. (1996) or advanced strategies suggested in Brucker et al. (1996, 1997)) and obtain integrated schedules of both raw materials and the robot's transportation operations as well as transportation and distribution of batches of semi-finished products.

In our study, we develop a new logistic and mathematical approach to increase the effectiveness of a military supply network. This is achieved by introducing a new optimization module, the "intermediary, or commanding, module", between the maintenance and transportation parts in order to manage to combine and coordinate the corresponding processes.

The measurement of the effectiveness of the integrated military supply networks is based on two performance measures: time of response and military effectiveness. To the extent these measures are improved, the entire military supply networks will be more effective. In the present work, we use a
version of the traditional job shop scheduling problem to describe the maintenance operations and employ the vehicle routing problem to present the transportation module (following the models by Dantzig and Ramser 1959 and Desrochers et al. 1988).

3. Demand-responsive military supply networks

3.1. Integrated approach to a military supply network

In accordance with the guidelines of the military standard MIL-STD (1990), we analyze two key modules in military training programs, maintenance and transportation. In terms of maintenance, once a confirmed order for repair parts is received, the maintenance unit assigns priorities to available batches of repair parts and attempts to schedule and carry deliver them as soon as possible (so that the batches with a higher priority will be processed first). At the end of the repair process, the fixed batches are delivered to their bases by employing the military vehicle fleet. The transportation detachment also prioritizes deliveries and executes all the prioritized shipments as soon as possible.

There are two basic approaches in modern management theory and practice. Both are well-known and widely used: (1) the "make-to-order" policy (i.e., once a confirmed order for products is received, products are produced, or, as in our study, repaired), and (2) the "demand-responsive transport" policy (offering customers the available transportation capacity in terms of vehicles and routes to satisfy their unpredictable-in-advance demands). Nevertheless, it is worth noting that the implementation of the sequential modular approach in solving integrated production-transportation problems may lead to obtain a local rather than a global optimum for the entire supply network. In order to properly analyze and create an effective military supply network, we suggest shifting from the sequential modular approach (Fig. 1a) to the hierarchical (integrated) approach (Fig. 1b).

The integrated approach expresses the integration of two basic concepts mentioned above: "make-to-order" and "demand-responsive transport". To improve the coordination between different components within the integrated supply network and to prevent the “domino effect” from arising in the first place, we introduce an additional module into the chain. Termed the "intermediary or commanding module" (IM), its main role is to reduce the discrepancies in time and common resources that are shared between different components. This module mimics the functions performed by the military commanding staff in modeling and managing integrated maintenance and transportation operations during military exercises. Therefore, the coordination between these operations should enhance the quality and effectiveness of army force training during military exercises, reducing operational costs and improving the utilization of resources.
We define and analyze two performance measures: time of response and military effectiveness. The first measure (time of response) describes the ability of the system to respond rapidly to the military requirements. If the latter requirement is sensitive to a particular choice of the performance measure depending on the application, then more than one performance measure can be associated with the time of response. In particular, customer waiting time (Barahona et al. 2007); requisition wait-time (Johnson and Levite 2003); and average customer wait-time (Gibson 2004) measure the elapsed time from the request to the delivery of the part to the end-customer. In this paper, we study another performance measure: the total waiting time between maintenance and transportation operations. This measure is appropriate for our model because the maintenance and transportation modules functioning independently provide the minimum time for performing their own operations, without taking into account the in-between time. For example, in situations in which the loading zone is vulnerable to enemy bombing or improvised explosive device (IED) attacks, the total waiting time between maintenance and transportation operations should be as short as possible. A reduction in this time period can reduce the number of soldiers killed.

In order to measure the ability of the integrated military supply network to provide the right product at the right time, we present and analyze the military effectiveness performance measure. This measure, which is the total amount of orders delivered to the military bases on time, is similar to the performance measures that can be found in Beamon (1999) (percent on-time deliveries).

4. Design of a new model for demand-responsive scheduling

4.1 Integrated maintenance-transportation model and algorithm

Transportation and maintenance modules that are operated independently and separately can only be used to determine a local optimum. This, in turn, leads to a coordination between them in terms of the common resources; time is usually not taken into account. Consequently, the effectiveness and demand-responsiveness of the entire supply network may not be ensured. In order to gain the advantages implied by coordinating the transportation and maintenance modules in terms of time and shared common resources, we introduce, as noted above, an intermediary module whose major role is
to consolidate the work between the above modules. We then expand the transportation-maintenance scheduling modules in two directions. First, we extend and develop each of the initial modules by introducing new variables and constraints in order to enhance their demand responsiveness. Second, along with solving the integer programming problems (IP) that have been obtained with an IP solver, we incorporate a heuristic procedure in which we combine several heuristic rules to help the decision-maker to obtain a good initial solution (a so-called “warm start”). Another important role of the heuristic procedure is to adjust several (excessive) variables in order to decrease the problem’s size.

The warm-start solutions generated by the heuristics in the three modules are used by the IP solver (GAMS) which works in two stages. First, it solves the IP problems formulated in the maintenance (MM) and transportation (TM) modules. The results are then delivered to the intermediary module (IM), where the GAMS solver solves an optimization problem formulated in this module. The output of the IM serves as the input for MM and TM, where the optimization problems are (again) solved by GAMS. The two stages are iteratively repeated until an acceptable solution at the intermediary module is obtained (see Fig. 2). The computational procedure is described in more detail in Section 6. A practical implementation of the suggested combination of the exact IP solver and the heuristic procedure is achieved within the ARENA 11.0 simulation system.

4.2 Intermediary module

The main role of the intermediary module is to discover parameters that were not taken into consideration when the modular approach was implemented. We associate these parameters with two major categories: time and capacity. The first category involves delay time parameters that occur between the MM and TM modules and are used to investigate and optimize response time. On the other hand, the capacity parameters express the effective utilization of all available military resources.

5. Mathematical models

The main purpose of the new models is to provide an optimal solution for the integrated scheduling of maintenance and transportation operations that have been obtained by coordinating the three modules: MM, TM and IM.
5.1 Maintenance module (MM)

The maintenance process starts when all the failed units are collected and then delivered from the military bases to the repair factory, where they are grouped into several batches according to their type. In our model the units of the same type (that is, of a certain batch) are to be processed on any machine successively, one after the other, without a break. Due to this, the set of units of any batch will be considered as a single job. Each job (batch) is characterized by a specific repair route, i.e., the sequence of the technological stages through which the job should pass. At any stage, a job can be processed by a number of alternative machines. The repairing of all jobs should be completed as soon as possible, that is, the total completion time is to be minimized. This model is well-known in scheduling theory as a classical job shop problem (see Pinedo 2002). We extend the job shop model by taking into account the following real-world constraints: (a) any job in the recovery plant can be processed by alternative, non-identical resources and (b) each job has its own due date. We introduce the following notation:

**Table 1:** Parameters of the MM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>The number of machines</td>
</tr>
<tr>
<td>$n$</td>
<td>The number of jobs (batches)</td>
</tr>
<tr>
<td>$i$</td>
<td>The machines, $i = 1,2,\ldots,m$</td>
</tr>
<tr>
<td>$j$</td>
<td>The jobs, $j = 1,2,\ldots,n$</td>
</tr>
<tr>
<td>$(i,j)$</td>
<td>Operation of processing job $j$ on machine $i$</td>
</tr>
<tr>
<td>$N$</td>
<td>Set of all operations, $N = {(i,j)}$</td>
</tr>
<tr>
<td>$A$</td>
<td>Set of routing constraints of the form $(i,j) \rightarrow (i',j)$. Such a routing constraint defines that operation $(i,j)$ should be processed on machine $i$ before being processed on machine $i'$</td>
</tr>
<tr>
<td>$K_j$</td>
<td>The number of the technological stages through which job $j$ passes within its maintenance process</td>
</tr>
<tr>
<td>$k(j)$</td>
<td>The technological stages through which job $j$ passes, $k(j) = 1,\ldots,K_j$</td>
</tr>
<tr>
<td>$L_{j,k(j)}$</td>
<td>Cluster (a group) of alternative machines available for job $j$ on stage $k(j)$. When a job $j$ is processed by a number of alternative machines at a stage $k(j)$, then these machines compose a cluster $L_{j,k(j)}$ such that only one machine of this cluster is selected at each stage</td>
</tr>
<tr>
<td>$p_{ij}$</td>
<td>Repair time of job $j$ on machine $i$</td>
</tr>
<tr>
<td>$D_j$</td>
<td>Due date for job $j$ to be repaired. The military commander’s staff assigns this parameter according to the extent and urgency of the maintenance activities required</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Cost of a completion time unit</td>
</tr>
</tbody>
</table>
**Decision variables:**

- \( y_{ij} \): Starting time of processing job \( j \) on machine \( i \).
- \( B_{ij,k(j)} \): Binary operation-machine variable: \( B_{ij,k(j)} = 1 \) if machine \( i \) is assigned to perform job \( j \) in stage \( k(j) \) (that is, \( i \in L_{j,k(j)} \)), and 0, otherwise. This variable is used to describe the ability of the job to be processed on alternative machines.

**Completion time for job \( j \):** \( F_j = \max \left( y_{ij} + p_{ij} \right) = y_{I(j),j} + p_{I(j),j} \), \( \forall j \), where \( I(j) \) is the index of the last machine in the last stage \( K_j \) of processing job \( j \).

**Total completion time:** \( \sum_{j=1}^{n} F_j \)

The scheduling problem is to select alternative machines and to determine the starting times of the operations in order to minimize the total completion time subject to the defined due dates and routing constraints. We suggest the following mathematical model for this problem:

**Problem MM:**

\[
\text{Min } \sum_{j=1}^{n} F_j \\
\text{subject to the constraints:}
\]

\[
y_{i'j} - y_{ij} \geq p_{ij}, \text{ for all } (i, j) \rightarrow (i', j) \in A \\
y_{ij} - y_{ij'} \geq p_{ij'}, \text{ or } y_{ij'} - y_{ij} \geq p_{ij}, \text{ for all } (i, j') \text{ and } (i, j), i = 1,2,...,m, j = 1,2,...,n
\]

\[
\sum_{i \in L_{j,k(j)}} B_{ij,k(j)} = 1, \text{ for all } L = \{L_{j,k(j)}\}, \ j = 1,2,...,n, \ k(j) = 1,2,...,K_j
\]

\[
F_j \leq D_j, \text{ for all } j = 1,2,...,n
\]

\[
y_{ij} \geq 0, \text{ for all } (i, j) \in N
\]

\[
B_{ij,k(j)} \in \{0,1\}, \text{ for all } i = 1,2,...,m, \ j = 1,2,...,n, \ k(j) = 1,2,...,K_j
\]

The optimization criterion is the minimization of total completion time (1). Inequalities (2) ensure that the operation \((i', j)\) cannot start before operation \((i, j)\) is completed. Inequalities (3) ensure that each machine can perform only one job at a time. Condition (4) ensures that only one alternative machine should be chosen at each stage (that is, in each cluster). Inequalities (5) express that the completion time for any job \( j \) is restricted by the due date. Inequalities (6) ensure that the decision
variables $y_{ij}$ are non-negative. Constraints (7) guarantee that the operation-machine assignment variables $B_{i,j,k(l)}$ are binary.

We observe that the considered maintenance scheduling problem is formulated as a mixed integer linear problem with disjunctive constraints. There are a significant number of software programs for solving such a type of problem. In this study, we use the standard MILP solver of the commercial optimization package GAMS (see Rosenthal 2010) to solve this problem. Results of computational experiments are presented in Section 7.

5.2 Transportation module (TM)
After the maintenance operation is completed, the military vehicle fleet consisting of the homogeneous trucks with the same loading capacity must transfer the repaired batches to the military bases. The size of the vehicle fleet involved in the transportation is assumed to be known; it is chosen by the military commander’s staff depending on logistical and security conditions. Also, in order to secure the transportation, armored vehicles can be used as a military convoy. Being accompanying by the convoy, the delivery trucks are able to employ shorter routes and move faster, and, therefore, they may reach the military bases in less time. Each base has its own demand and should be served by exactly one vehicle within the specified time windows. The problem to be solved is to minimize the following cost components: (i) the total travel cost associated with transporting repaired batches from the repair plant to their final destinations (by the trucks and the accompanying military convoy), and (ii) the total cost of loading/unloading operations for all delivered batches. The considered problem is a version of the vehicle routing problem (VRP). The operations research literature contains many versions of the VRP and a significant number of computational methods for their solution (see, for example, the excellent survey by Irnich et al. 2014). In spite of an abundant literature on the VRP, to the best of our knowledge, there is no model comprehensively treating the military logistics factors, which we intend to consider in the present paper. Our model is similar to the VRP models by Dantzig and Ramser (1959), Laporte et al. (1985) and Desrochers et al. (1988), however, in contrast to the model given in Dantzig and Ramser (1959), our model takes the time windows and truck capacity constraints into account; further, in comparison with the models in Laporte et al. (1985) and Desrochers et al. (1988), we introduce two types of vehicles (delivery trucks and armed convoy) involved in the military logistics; and, finally, none of the above works handle corresponding transportation and loading costs. We introduce the following notation:
Table 2: Parameters of the TM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>Graph, $G = (V, A)$</td>
</tr>
<tr>
<td>$V$</td>
<td>Set of vertices, $V = {0} \cup \mathcal{N}$, where ${0}$ indicates the depot (repair factory) and $\mathcal{N} = {1, 2, \ldots, n}$ is the set of military bases</td>
</tr>
<tr>
<td>$A$</td>
<td>Set of arcs, $A = ({0} \times \mathcal{N}) \cup I \cup (\mathcal{N} \times {0})$, where $I \subset \mathcal{N} \times \mathcal{N}$ is the set of arcs connecting the military bases, ${0} \times \mathcal{N}$ contains the arcs from the depot to the military bases, whereas $\mathcal{N} \times {0}$ contains the arcs from the military bases to the depot</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Demand at military base $i$, $i \in \mathcal{N}$</td>
</tr>
<tr>
<td>$K$</td>
<td>The truck fleet size</td>
</tr>
<tr>
<td>$C$</td>
<td>Capacity of a truck</td>
</tr>
<tr>
<td>$t_{ij}$</td>
<td>Travel time from military base $i$ to $j$. We assume that the transportation time is symmetric, that is, $t_{ij} = t_{ji}$ and $t_{ii} = 0$</td>
</tr>
<tr>
<td>$\Delta t_{ij}$</td>
<td>Time shift, that is, the maximal possible decrease in the transportation time $t_{ij}$ associated with employing a military escort to secure the transportation over an arc $(i, j)$</td>
</tr>
<tr>
<td>$u_i$, $r_i$</td>
<td>Time windows at base $i$ (lower and upper time bounds, respectively)</td>
</tr>
<tr>
<td>$c_{ij}$</td>
<td>Travel cost associated with an employing truck for transportation from $i$ to $j$</td>
</tr>
<tr>
<td>$b_{ij}$</td>
<td>Travel cost associated with employing a military convoy for securing transportation from $i$ to $j$</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>Loading cost at military base $i$</td>
</tr>
<tr>
<td>$M$</td>
<td>Any very large number</td>
</tr>
</tbody>
</table>

**Decision variables:**

- $s_i$ - Arrival time at military base $i$, $s_0 = 0$.
- $y_i$ - Load of truck arriving at military base $i$, $y_0 = 0$.
- $x_{ij}$ - Truck binary routing variable: $x_{ij} = 1$ if arc $(i, j)$ is used on the tour by a truck, and 0, otherwise.
- $z_{ij}$ - Convoy binary routing variable: $z_{ij} = 1$ if arc $(i, j)$ is used on the tour by a convoy, and 0, otherwise.

The transportation problem is to obtain optimal routes to minimize total travel cost associated with employing trucks and armed escort in the delivery process $\sum_{(i,j) \in A} [c_{ij}x_{ij} + b_{ij}z_{ij}]$ as well as the total
loading cost $\sum_{i \in N} \alpha_i y_i$ subject to the constraints on the bounded vehicle fleet, time windows and truck capacity. We propose the following mathematical model for this problem:

**Problem TM.**

$$\text{Min } \sum_{(i,j) \in A} [c_{ij} x_{ij} + b_{ij} z_{ij}] + \sum_{i \in N} \alpha_i y_i$$

subject to the constraints:

$$\sum_{j \in N} x_{0j} \leq K$$

$$\sum_{i \in N} x_{i0} = \sum_{j \in N} x_{0j}$$

$$\sum_{j \in N} x_{ij} = 1, \forall i \in N$$

$$\sum_{j \in N} x_{ji} = 1, \forall i \in N$$

$$z_{ij} \leq x_{ij}, \forall (i, j) \in A$$

$$s_i + t_j - \Delta t_{ij} z_{ij} - s_j \leq M \cdot (1 - x_{ij}), \forall (i, j) \in I \cup \{0 \times N\}$$

$$u_i \leq s_i \leq r_i, \forall i \in N$$

$$y_i + d_j - y_j \leq M \cdot (1 - x_{ij}), \forall (i, j) \in I \cup \{0 \times N\}$$

$$0 \leq y_i \leq C, \forall i \in N$$

$$s_i, y_i \geq 0, \forall i \in N$$

$$x_{ij}, z_{ij} \in \{0,1\}, \forall (i, j) \in A$$

The optimization criterion is given in (8). Inequalities (9) ensure that at most $K$ trucks depart from the depot. Equation (10) ensures that all trucks departed from the depot also have to return. Equations (11) and (12) express that each military base must be served exactly once. Inequalities (13) ensure that the military convoy cannot be employed on arc $(i, j)$ if this arc is not used by a truck on the tour. Inequalities (14) ensure that a truck cannot arrive to military base $j$ before $s_i + t_{ij}$, if it travels from $i$ to $j$. Inequalities (15) ensure that all time windows are respected. Conditions (16) and (17) ensure the feasibility of the loads. Condition (18) expresses that the decision variables $s_i, y_i$ are non-negative. Finally, condition (19) defines the binary variables.

Inequalities (14) and (16) provide the subtour elimination in the same way as in the model by Desrochers et al. (1988). Similarly to the description of the MM module, we again use the standard MILP solver of GAMS for its solution. The computational results are presented in Section 7.
5.3 Intermediary module (IM)

We define the main role of the intermediary module as a coordination between MM and TM operations in terms of capacity (effective usage of the military resources) and time (reducing delay time that occurs between the MM and TM). In the IM, we design and solve the following optimization problem: determining the best batch-vehicle allocations such that the total waiting time between the maintenance and transportation operations will be minimized. We introduce the following notation:

Table 3: Parameters of the IM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>The trucks, $k = 1, ..., K$</td>
</tr>
<tr>
<td>$\gamma_j$</td>
<td>Allowed delay time that elapses between the moment when repair of batch $j$ is finished and the moment when its transportation starts</td>
</tr>
<tr>
<td>$[TD', TD'']$</td>
<td>Interval of allowed values for the delay time between all maintenance and transportation operations; this parameter is assigned by the military commander’s staff depending on the urgency of the part needed</td>
</tr>
<tr>
<td>$TQ_j$</td>
<td>The entire number of units of batch $j$ that has to be delivered to the customers</td>
</tr>
<tr>
<td>$ST_{kj}$</td>
<td>Starting time of transportation for batch $j$ by truck $k$ (this parameter is received as the output of the transportation module).</td>
</tr>
<tr>
<td>$F_j$</td>
<td>Completion time for batch $j$ (this parameter is received as the output of the maintenance module).</td>
</tr>
</tbody>
</table>

Decision variables:

$\bar{U}_j$ - Finish time shift, that is, the increase in the finish time of a repair operation of batch $j$ made for reducing the total waiting times between the repair and transportation operations.

$\bar{U}_k$ - Starting time shift, that is, the increase/decrease in the starting time of a transportation operation of truck $k$.

$Q_{kj}$ - Number of repaired units of batch $j$ that are loaded to truck $k$.

Problem IM.

$$\text{Min} \sum_{j=1}^{n} \sum_{k=1}^{K} [ST_{kj} + \bar{U}_k - (F_j + \bar{U}_j)]$$

(20)

$$ST_{kj} + \bar{U}_k - (F_j + \bar{U}_j) \leq \gamma_j, \text{ for } j = 1, ..., n, k = 1, ..., K$$

(21)

$$TD' \leq \sum_{j=1}^{n} \bar{U}_j + \sum_{k=1}^{K} \bar{U}_k \leq TD''$$

(22)

$$\sum_{j=1}^{n} Q_{kj} \leq C, \text{ for } k = 1, ..., K$$

(23)

$$\sum_{k=1}^{K} Q_{kj} = TQ_j, \text{ for } j = 1, ..., n$$

(24)
\[ \bar{U}_j, \bar{U}_k \in \mathbb{R}, \text{ for } j = 1, \ldots, n; k = 1, \ldots, K \]  

\[ Q_{kj} \geq 0, \text{ for } j = 1, \ldots, n, k = 1, \ldots, K \]  

According to (20), the objective is to minimize the total waiting time between the maintenance and transportation operations. Inequalities (21) ensure that the delay time between the instant when repair of batch \( j \) is finished (\( F_j \)) and the instant when its transportation to the customer (i.e., the base) by truck \( k \) starts (\( ST_{kj} \)) is limited. Inequalities (22) ensure that the total time gap between all the maintenance and transportation operations is limited. Inequalities (23) ensure that the total amount of batches delivered by truck \( k \) does not exceed the capacity \( C \). Equations (24) ensure that all the required quantity \( TQ_j \) of units in batch \( j \) is delivered to the end-customers. Condition (25) expresses that the variables \( \bar{U}_j, \bar{U}_k \) are real numbers. Finally, condition (26) expresses that the variables \( Q_{kj} \) are non-negative. The IM is formulated as the MILP and we solve it by the standard MILP solver of the GAMS. The mode of cooperation and iterative data exchange between blocks was described in Section 4.1.

6. Description of the algorithm

As a solution to the integrated transportation-maintenance scheduling problem, we present a heuristic search procedure that combines different heuristic scheduling rules. At first, the heuristic rules are implemented using ARENA 11.0. As a result, good feasible solutions are found and some of the (integer) variables become fixed. This decreases the problem size. The main fixed variables are the starting time \( y_{ij} \) of processing job \( j \) on machine \( i \) (in the module MM) and the truck binary routing variable \( x_{ij} \) (in the module TM). The variables become fixed if during the prescribed number of iterations their values remain the same. The solutions that emerge are used as starting solutions for solving the integer programs (IP) in the MM, TM, and IM modules. The regular integer programming solver GAMS/CPLEX (see Rosenthal 2010) is used to achieve this objective.

Each entity in the simulation model is associated with either a batch in the MM or a vehicle in the TM. Each maintenance batch in MM is processed following a prescribed path consisting of different types of repair machines with the given due dates being taken into account. The best processing policy is determined by using and comparing various heuristics rules, namely: EDD, LIFO, FIFO, and slack-based priorities. The description of these basic heuristic rules can be found in Pinedo (2002), and is not discussed here.

At the end of the maintenance process, the fixed batches are delivered back to their bases by using a military fleet consisting of vehicles with different capacities. These capacities are generated randomly at each iteration by using the following rule: \( \text{New}_\text{Capacity} = \text{Uniform}[0.5, 1]*\text{Initial}_\text{Capacity} \). In turn, in TM we use the coefficient weighted time distance (CWTD), a
heuristic procedure proposed by Galic et al. (2006). The site with the lowest decision measure will be served first. We extend Galic's algorithm as follows: each parameter in the heuristic algorithm (namely, time, distance and amount) is assigned a specific weight that is updated randomly at each iteration according to the uniform distribution.

The role of the IM is to coordinate the operations between MM and TM. The IM is presented in the simulation model in terms of: (a) an existing, additional pool of vehicles provided by the external third-part logistic companies; (b) determining new due dates for the jobs in MM and new starting times for the transportation operations of the vehicles with respect to the time of response and military effectiveness performance measures and (c) updating the vehicle capacity. The iterative solution procedure is terminated when the improvement in the value of the objective function in the IM is less than a prescribed given value.

The following flowchart in Fig. 3 visually presents the algorithm:

![Flowchart of the algorithm](image)

**Fig. 3.** The flowchart of the algorithm.
7. Computational results

In the operations research literature, there are two distinct approaches for modeling integrated scheduling and logistics systems. The first approach, “monolithic”, is based on grouping all the conditions of several sub-problems into a single problem and formulating the latter as a large mixed-integer program to be solved to optimality (Sawik 2009, Steinrücke 2011). The second approach, referred to as modular or hierarchical, partitions a large-scale problem into a network of linked sub-problems and optimizes them sequentially so that an output of one problem serves as an input for another one. Then the computational time is considerably reduced although the obtained solution is sub-optimal (Sawik 1999, Park 2005). We carried out two types of computational experiments to determine, first, whether or not our hierarchical approach was effective in comparison with the exact monolithic model and, second, the efficiency of the sequential modular model and our hierarchical model.

7.1 Comparison of monolithic and heuristic approaches

In order to estimate the efficiency of the exact monolithic approach, we constructed an induced parametric integer programming problem in which all the constraints of Problems MM, TM and IM of Section 5 are grouped together into one monolithic problem. Since the structure of this problem directly follows from the problems indicated, we do not need to present here its formulation.

To find an exact solution of the monolithic problem, we used the solver GAMS/CPLEX. This solver is based on the branch-and-bound enumerative search and designed for solving non-linear mixed integer problems. The comparison was carried out on the NEOS server. The hardware specifications of the server machines (NEOS-2 and NEOS-4) are as follows: Dell PowerEdge R410 servers, CPU - 2x Intel Xeon X5660 @ 2.8GHz (12 cores total), HT Enabled, 64 GB RAM. The problem instances were generated randomly as shown in Table 4.

Table 4

Data used in the comparison analysis.

<table>
<thead>
<tr>
<th>Data type</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of jobs</td>
<td>Unif(5,24)</td>
</tr>
<tr>
<td>Number of machines</td>
<td>Unif(2,4)</td>
</tr>
<tr>
<td>Number of vehicles</td>
<td>Unif(2,3)</td>
</tr>
<tr>
<td>Number of bases</td>
<td>Unif(2,3)</td>
</tr>
<tr>
<td>Repair time (in hrs)</td>
<td>Unif(0.1,0.3)</td>
</tr>
<tr>
<td>Transportation time (in hrs)</td>
<td>Unif(0.2,1)</td>
</tr>
<tr>
<td>Time windows (lower bound)</td>
<td>Unif(2.5,4)</td>
</tr>
<tr>
<td>Time windows (upper bound)</td>
<td>Unif(4.5,6)</td>
</tr>
<tr>
<td>Service time (in hrs)</td>
<td>0.1</td>
</tr>
</tbody>
</table>

A total of about 200 different instances were solved. For all experiments, the calculation time was limited to 1 CPU-hour per problem instance. For each instance, the solver provided an exact but computationally expensive solution. The largest problem instance for which we could find an exact
solution using the GAMS solver had 82 jobs grouped into 9 batches; 10 machines; 6 vehicles; and 14 customers. The induced monolithic Mixed Integer Programming Problem had 350 variables, among them 280 discrete ones and contained 580 constraints. For this instance, the running time was 52 minutes. The solution of smaller instances required from eight to 50 minutes. To solve all these instances, the GAMS/CPLEX via NEOS server required a total of 85 hours of CPU time.

In addition, all these problems were also solved by the ARENA 11.0 that spent from 50 seconds to ten minutes per instance. All the derived solutions turned out to be optimal. The computational experiment showed that for the problem instances that were considered, the heuristic system yielded the exact solution and, operated much faster than the monolithic one. These computational results qualitatively agree with the simulations reported earlier by Sawik (1999), Sawik (2009).

7.2 Comparison of sequential and hierarchical approaches

In this sub-section, we compare numerically the sequential and hierarchical approaches. In order to do this, we performed experiments in the simulation model with random data using two scenarios: (1) the amount of jobs ranged from 25 to 200 and (2) there were two to twenty vehicles (the corresponding data and computational details can be provided by the authors upon a request). The output consisted of the total waiting time between the maintenance and transportation operations and the total number of jobs delivered to end-customers on time. The computational results for the two scenarios are as follows:

(a) Scenario 1. The number of jobs rose from 25 to 200. Fig. 4 presents the time of response as a function of the number of jobs. Fig. 5 presents the military effectiveness measure as a function of the number of jobs.

![Fig. 4. The time of response as a function of the number of jobs.](image-url)
The graph in Fig. 4 shows that the integrated approach yields a better time of response than the sequential modular approach. With an increase in the number of jobs, the total waiting time in the integrated approach grows more slowly. The reduction in the waiting time is achieved by a better assignment of the jobs to the available vehicles. The obtained results are shown in Fig. 5, where we can see that with the increase in the number of jobs, the integrated approach gives a better jobs-on-time/total-number-of-jobs ratio.

(b) Scenario 2. The number of vehicles increased from 2 to 20. Figs. 6. and 7, respectively, present the time of response and military effectiveness as functions of the number of vehicles.

Fig. 5. The military effectiveness measure as a function of the number of jobs.

Fig. 6. The time of response as a function of the number of vehicles.

Fig. 7. The military effectiveness as a function of the number of vehicles.
The graphs in Fig. 6 show that the integrated approach achieves a better time of response performance measure than the modular approach. From Fig. 7, we can see that the integrated approach achieves a higher military effectiveness than the modular approach. With an increase in the number of vehicles in the model, the number of jobs delivered on time to the end-customers in the integrated approach increases more rapidly. The reason of this is that the IM uses the available vehicle fleet more effectively so that the number of jobs delivered in time to the end customers is larger than in the standard modular approach not using IM. In both scenarios the average running time of ARENA 11.0 was between two and 53 minutes.

8. Conclusions

Projecting and sustaining a military force, particularly during training exercises, hinges on the successful establishment and management of demand-responsive military supply networks. The large number of different types of items in military warehouses, the tight constraints on operational delays and changeable time demands are the main factors that influence the responsiveness of military supply networks. For repair and maintenance operations, it is unrealistic to rely exclusively on the local optimization of maintenance and transportation operations.

The responsiveness of the military supply network is formally described and analyzed with the help of two performance measures: time of response and military effectiveness. A high demand responsiveness is obtained by integrating the maintenance and transportation modules. Integrating these separate modules involves incorporating an intermediary module. This module includes a number of parameters that are not taken into account when the repair and transportation processes are planned separately.

The role of the intermediary module is twofold. On the one hand, this module interprets and mimics the role of the military commanding staff in effectively modeling and managing integrated maintenance and transportation operations during military exercises. An effective modeling and managing make military supply networks more demand-responsive and effectual. On the other hand, the intermediary module makes it possible to achieve a global mathematical optimum rather than a local optimum for a military supply network.

To find an optimal solution of the integrated mathematical model, we introduced a new two-stage heuristic search method that we used to solve at the first level two scheduling problems arising in the independent maintenance and transportation modules. Then, at the second level, we use the outputs of the two modules, obtained at the first level, as the input for the integrated problem of the second level. The optimal solution obtained at the second level is either accepted or, if necessary, leads to a change of the input data of the modules at the first level. This algorithm is solved by a combination of heuristic rules and mixed integer programming techniques.
In order to compare the efficiency of the exact and heuristic approaches, an equivalent monolithic integer programming (IP) problem was formed. A set of randomly generated IP problems was solved sequentially by the branch-and-bound method implemented by the standard software (GAMS/CPEX) and by our heuristic method which was implemented with the ARENA simulation system. The exact method proved to be computationally expensive, its running time being prohibitively larger even for small size problems.

The comparison across the sequential modular and integrated approaches was based on measuring the time of response and military effectiveness. The numerical results demonstrate that the integrated approach achieves better military performance measures than the modular one. Coordinating the scheduling decisions between different participants improves the effectiveness of both the individual participants as well as the entire military supply network. These observations confirm those that have been made since the 1990s in many other studies on supply chain management (see, e.g., Dhaenens-Flipo and Finke 2001 and Kreipl et al. 2006, among others). Note that the mathematical models described in this paper, although being initiated by military logistics applications, can also be used in different non-military commercial supply networks. A promising direction for future research would be to extend the suggested two-stage modeling approach for exploring multi-echelon supply networks.

References


