Proactive scheduling and reactive real time control in

Industry 4.0 manufacturing systems

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Abstract

Scheduling in Industry 4.0 systems belongs to a class of problems that have mixed structural-temporal-logical constraints. In other words, a strong coupling is considered when product and process are created at the same time. As a result of the proven NP-hardness of such problems, solution methods have extensively utilized different decomposition principles. The known decomposition methods in discrete optimization are founded on the difficulties in deriving analytical properties. The existing solutions in continuous optimization are based on the maximum principle and use a dynamic process decomposition and the natural logic of time. By combining the advantages of continuous and discrete optimization, this paper develops a decomposition method for shop floor scheduling in Industry 4.0 manufacturing systems. Technically, this study proposes to decompose dynamically the large-scale assignment matrix according to the precedence relations between the operations of the jobs and considers only the operations that satisfy these precedence relations at a given time point in small-dimensional, discrete optimization models. Continuous optimization is used to generate a schedule from the assignments found in the discrete optimization models at each time point by extremizing the Hamiltonian function at this time point subject to scheduling objective(s). In addition, the execution of the operations in time can be accurately modeled in continuous time as a continuous state variable; the machine availability and capacity disturbances at the machines are also considered. The method developed provides further insights into decomposition methods for scheduling and is supported by an analytical analysis and an algorithmic realization.

Keywords: scheduling, Industry 4.0, flexible flow shop, manufacturing, optimal control, algorithm, real-time scheduling, dynamic scheduling

MSC classification: 90B30, 49J15, 90B35

1. Introduction

Individualization of products is a critical business capability and requires flexible and customized production systems. Because of the increased complexity of flexible, small batch manufacturing, the costs of individualized production are typically higher than in mass production systems. The Industry 4.0 technology has enabled new production strategies, particularly through the use of cyber-physical systems that require highly customized assemblies (Erol et al. 2016, Oesterreich and Teuteberg 2016, Kumar et al. 2016, Nayak et al. 2016, Battaïa et al. 2017a, Hwang et al. 2017). The ultimate objective of these systems is to facilitate a flexible customized manufacturing at the lower cost of mass production.

Such innovative production strategies engender new challenges and opportunities for short-term job scheduling and sequencing. In particular, Kusiak (2018) points out the issue of strong coupling in smart manufacturing when product and process are created at the same time. Simultaneous product and process creation results in a class of scheduling problems that have mixed structural-temporal-logical constraints with order scheduling based on a search for free resources for free operations (Dolgui et al. 2018). Manufacturing processes for different customer orders may have individual structures of the stations such that the flexible stations are able to execute different functions subject to individual sets of operations within the jobs (Weyer et al. 2015, Ivanov et al. 2016a,b, Nayak et al. 2016, Battaïa et al. 2017b, Zhong et al. 2017). Therefore, an integrated problem of simultaneous, structural-functional synthesis of the Industry 4.0 customized assembly systems and job scheduling in these systems arises.

In the given problem class, multi-stage, flexible, job-flow shop scheduling problems with flexible machines have been studied (Ivanov et al. 2016a,b, Bozek and Werner 2017). Kyparis and Koulamas (2006) considered a multi-stage, flexible flow shop scheduling problem with uniform parallel machines at each stage and makespan minimization. This study proposed a heuristic algorithm for this strongly NP-hard problem. Tazar et al. (2006) considered a scheduling problem for a set of independent jobs with sequence-dependent setup times and job splitting on a set of identical parallel machines such that the maximum completion time (i.e., the makespan) is minimized. For this NP-hard problem, the study developed a heuristic algorithm using linear programming (LP). Furthering these insights, Bozek and Werner (2017) developed an optimization method for flexible job shop scheduling with lot streaming and sublot size optimization. It can be noted that a review of solution techniques for flexible (or hybrid) flow shop problems has been given e.g. in the paper by Ruiz and Vazquez-Rodriguez (2010), and a review on flexible job shop scheduling problems has been given by Chaudry and Khan (2016).

In light of the proven NP-hardness of such problems, solution methods for the simultaneous structural-functional synthesis of customized Industry 4.0 assembly systems and job scheduling in these systems need to be developed using different decomposition principles. This is because large-scale MILP models would be negatively influenced by the computational complexity of these problems. The known solutions in discrete optimization based on decomposition, such as data-driven or clustering approaches (Chen et al. 2013a,b), are founded on the difficulties in deriving analytical properties.

The existing solutions in continuous optimization based on the maximum principle and the control of a dynamic system use a decomposition of a dynamic process and the natural logic of time (Ivanov and Sokolov 2012). These solutions were primarily applied to technical systems (e.g., space shuttle movement control) and rely on a proven analytical axiomatic of optimal control. In the 1990s, optimal control models based on the maximum principle were applied to master production scheduling, but they did not consider the precedence relations within the jobs (Dolgui et al. 2018) and focused mostly on small-dimensional problems (Kogan and Khmelnitsky, 2000).

The major intention of this paper is to provide further insights into scheduling in smart manufacturing with simultaneous product and process creation using decomposition methods by combining the advantages of continuous and discrete optimization. In particular, we focus on shop floor scheduling in very flexible manufacturing systems such as Industry 4.0. Technically, this study proposes a dynamical decomposition of a large-scale assignment matrix according to the precedence relations between the operations of the jobs and considers only the operations that satisfy the precedence relations at the given time point in small-dimensional LP models. Discrete optimization algorithms (B&B, Hungarian method) are used for scheduling in these matrices of small dimension at each time point. Continuous optimization algorithms (e.g., the method of successive approximations, or the method of Krylov-Chernousko, see Ivanov et al. 2016 and Dolgui et al. 2018, and the references in these papers) are used to create a schedule from the LP results generated at each time point by extremizing the Hamiltonian function at this time point subject to some criteria (e.g., tardiness). In addition, the execution of the operations in time can be accurately modeled in continuous time as a continuous state variable, while considering machine availability and capacity disturbances at the machines.

The remainder of this paper is as follows. Sect. 2 is devoted to a verbal problem statement. In Sect. 3, the research methodology is presented. Sect. 4 develops generalized models of selecting the design of the manufacturing process and operation sequencing. It also considers a model for a simultaneous process design and sequencing. In Sect. 5, the generalized model from Sect. 4 is modified regarding a flexible dynamic scheduling in Industry 4.0 systems. Subsequently, a computational algorithm is presented. Sect. 6 concludes the paper by summarizing the most important results of this study and outlining some issues for future research.

2. State of the art

Practical environments for applying scheduling and sequencing models and algorithms to a simultaneous structural-functional synthesis of the customized assembly system are multifaceted. With the help of smart sensors and plug-and-produce cyber-physical systems, the stations in the assembly system are capable of changing the operation processing and setup sequences according to the actual order of the incoming flows and capacity utilization (Otto et al. 2014, Theorin et al. 2017). In the FOUP (front opening unified pods) technology in the semiconductor industry, robots are used for a real-time operation sequencing. Robots read the information about the products from the sensors and tags and decide flexibly, where to forward a wafer batch next (Mönch et al. 2012).

The recent literature has included a variety of principles and approaches to the design and scheduling of flexible and reconfigurable assembly systems with a focus on balancing, scheduling, and sequencing (Boysen et al. 2007, Chaube et al. 2012, Delorme et al. 2012, Battaïa and Dolgui 2013, Battaïa et al. 2017). In these studies, models and methods have been presented for solving problems related to the optimization of the performance intensity of an assembly system for sets of flexibly intersecting operations.

For systems which consider both the machine selection for each part of the manufacturing process and the loading sequences of the parts to the machines, Blazewicz et al. (2001) showed that these problems are NP-hard. In particular, scheduling with *alternative parallel machines* addresses the practical challenge that at each stage of the manufacturing process, alternative machines may execute the operations. This creates flexibility in the process plan and requires both a machine assignment and sequencing of tasks (Yu et al. 2011, Janiak et al. 2013, Blazewicz et al. 2015). In practice, the optimization objectives consider throughput maximization, lateness minimization, and equal machine utilization.

Józefowska et al. (2002) presented a heuristic approach to allocate a continuous resource in discrete-continuous scheduling problems to minimize the makespan. Kyparis and Koulamas (2006) considered a multi-stage, flexible flow shop scheduling problem with uniform parallel machines at each stage and the minimization of makespan. Tahar et al. (2006) considered the problem of scheduling a set of independent jobs with sequence-dependent setup times and job splitting on a set of identical parallel machines such that the maximum completion time (makespan) is minimized.

During the last three decades, a variety of papers presented results and algorithms for flexible flow shop and job shop scheduling. The paper by Ribas et al. (2010) first classifies the papers for flexible (hybrid) flow shop scheduling according to the production characteristics and limitations and then according to the solution approaches proposed. The reader can find 164 references to papers dealing with hybrid flow shop problems. In parallel, another review on solution approaches with 225 references has been presented by Ruiz and Vazquez-Rodriguez (2010). For flexible job shop scheduling problems, a recent review has been given by Chaudry and Khan (2016). The authors found 404 papers dealing with flexible job shop problems from the period 1990 - 2014. The interested reader can also find 212 cited references in this paper. In a very recent paper, Shen et al. (2018) presented a mathematical model and a tabu search algorithm for the flexible job shop problem with sequence-dependent setup times and minimizing the makespan. The results show that their algorithm outperforms most existing approaches for the classical flexible job shop problem.

Lauff and Werner (2004), Jungwattanakit et al. (2009), Sotskov et al. (2013), Choi et al. (2013), Harjunkoski et al. (2014), Bożek and Wysocki (2015) have pointed out that specific scheduling problems require further investigation and the application of a broad range of methodical approaches. Control approaches to scheduling are of vital importance for addressing the flow dynamics in the assembly line. The studies by Sarimveis et al. (2008), Ivanov et al. (2012), and Harjunkoski et al. (2014) showed a wide range of advantages regarding the application of control-theoretic models in combination with other techniques for scheduling in manufacturing. These advantages include, but are not limited to, a non-stationary process view and the accuracy of continuous time.

Optimal control approaches provide a different perspective than mathematical programming methods. Optimal control approaches represent schedules as trajectories. The first studies in this area were devoted to inventory control. One of these studies (Eilon 1961) was published in the first volume of the *International Journal of Production Research* (*IJPR*). Later, Hwang *et al*. (1967, 1969) were among the first to apply optimal control and the maximum principle to multi-level and multi-period master production scheduling which determined an optimal control (i.e., production) for the corresponding state (i.e., the inventory trajectory). Albright and Collins (1977) developed a Bayesian approach to the optimal control of continuous industrial processes. Bedini and Toni (1980) developed a dynamic model for the planning of a manufacturing system. The maximum principle was used to formulate the problem and to obtain a solution. Flexible manufacturing systems and their dynamics have been examined in numerous studies (e.g., Stecke and Solberg 1981). The stream of production scheduling was continued by Kimemia and Gershwin (1985), Kogan and Khmelnitsky (1996), and Khmelnitsky *et al*. (1997), who applied the maximum principle in discrete form to the planning of continuous-time flows in flexible manufacturing systems

Applications of optimal control to scheduling problems are encountered in production systems with single machines (Giglio 2015), job sequencing in two-stage production systems (Lou and Van Ryzin 1989, Sethi and Zhou 1996), and multi-stage machine structures with alternatives in job assignments and intensity-dependent processing rates, such as those in flexible manufacturing systems (Sharifnia et al. 1991, Maimon et al. 1998, Yang et al. 1999, Ivanov and Sokolov 2013a, Pinha et al. 2015), supply chains as multi-stage networks (Ivanov and Sokolov 2012, Ivanov et al. 2013), and Industry 4.0 systems based on a data interchange between the product and stations, flexible stations dedicated to various technological operations, and real-time capacity utilization control (Ivanov et al. 2016a).

In previous studies the selection of the process structure and the respective station functionality for the execution of the operations were considered in isolation. In many real life problems, such an integration can have a significant impact on the process efficiency (Bukchin and Rubinovitz, 2003). The problem of a simultaneous structural-functional synthesis of the customized assembly system is still at the beginning of its investigation (Levin et al. 2016).

Optimal control scheduling models with only terminal constraints typically address the level of master production scheduling (Hwang et al. 1967, 1969, Kimemia and Gershwin 1983, Jiang and Sethi 1991, Khmelnitsky et al. 1997, Kogan and Khmelnitsky 2000). Scheduling models with both terminal and logical constraints can also be applied to flow shop and job shop scheduling (Kalinin and Sokolov 1985, 1987, Ivanov and Sokolov 2013a, Ivanov et al. 2016a,b) as well as to supply chain scheduling (Ivanov and Sokolov 2012, Ivanov et al. 2013).

Previously isolated insights gained in hybrid shop scheduling, scheduling and sequencing with alternative parallel machines, and optimal control scheduling models with both terminal and logical constraints can now be integrated into a unified framework of Industry 4.0 and must be extended towards models with hybrid structural-terminal-logical constraints (Dolgui et al. 2018). The three most important prerequisites for such an integration, i.e., the data interchange between the product and stations, flexible stations dedicated to various technological operations, and a real-time capacity utilization control, are enabled by the Industry 4.0 technology.

This study extends previous publications of the authors (Ivanov et al. 2016a,b): the problem statement and the scheduling model consider the structural synthesis and sequencing decisions of the manufacturing process. In the studies (Ivanov et al. 2016a,b), only scheduling decisions were considered and the design of the process structure was assumed to be fixed, i.e., the design of a flow shop process was considered.

This study develops an optimal control model for the simultaneous structural-functional design of a customized manufacturing process and the sequencing of the operations within the jobs. The developed theoretical framework presents a contribution to flexible scheduling in the emerging field of Industry 4.0-based innovative production systems.

3. Model

3.1. Problem statement

Consider a customer system which interacts with an assembly system. The customer system generates orders (jobs) each of which has an individual sequence of the technological operations. The interaction of the customer and the assembly systems results in alternatives for the design of the manufacturing process. Consider an assembly system composed of partially uniform stations which are able to execute some technological operations. Each station has multiple channels, each of which is dedicated to a set of technological operations. Since multiple stations may perform the same operations, alternatives for job scheduling and sequencing exist which are subject to actual capacity utilization, machine availability, and time- and cost-related parameters (Fig. 1).

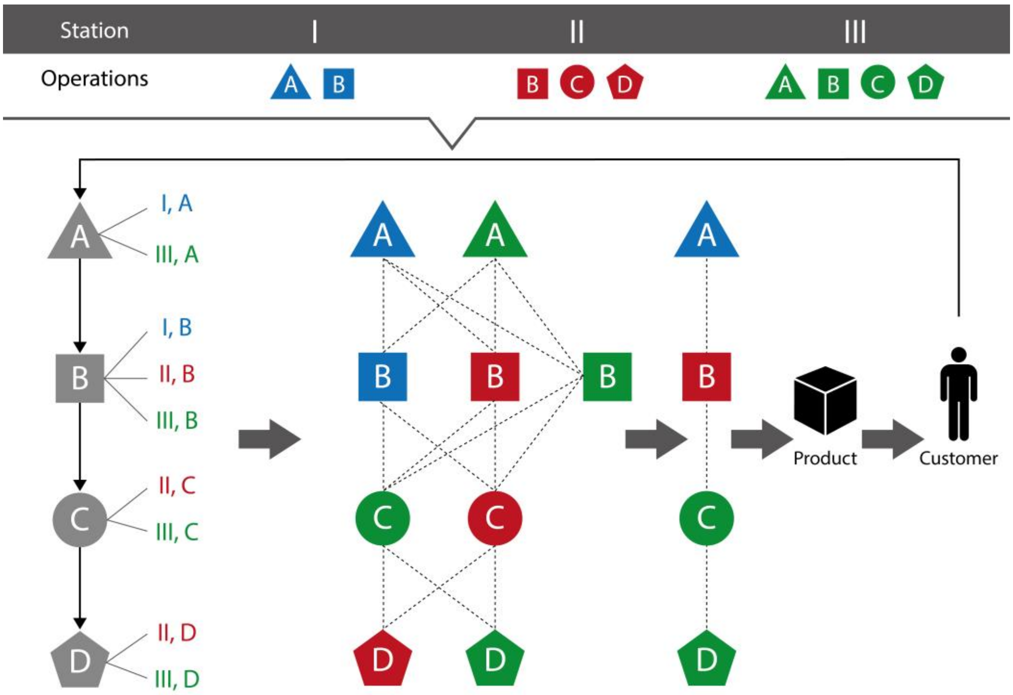

Fig. 1. Interaction of the customer and the assembly systems

In Fig. 1, a customer order (i.e., a job) is considered which contains the four operations A, B, C, and D. Three flexible workstations (i.e., machines) are considered. Station I is capable to process operations A and B, station II is capable to process operations B, C, and D, and station III is capable to process all operations A, B, C, and D. The processing of an operation at different workstations differs in regard to time and costs. Each station is capable of handling only one operation at a time subject to the availability of a non-stationary station at each stage (i.e., “availability windows”), which is expressed in the preset matrix time function. Once a job is forwarded to the assembly system, the operations of this job can be executed at some alternative stations. Therefore, transportation in-between the stations can be considered as an alternative graph.

In generalized terms, the problem considered captures the following features. The processing speed of each machine is described as a time function and is modeled by material flow functions (integrals of processing speed functions) and the resulting processing time of the operation is, in general, dependent on the characteristics of the processing channel. The processing and routing capacities are constrained. Setups are included into the analysis. The lot-sizes are fixed and known in advance. The temporary unavailability of capacity as a consequence of possible disruptions is included. Material supply and consumption dynamics are considered. The optimization is performed subject to the following performance indicators (control functionals): throughput, lead-time, makespan, total lateness, equal utilization of the stations in the assembly line, and waiting time. The *first task* is to design the manufacturing process, i.e., to assign the operations to stations at each stage of the technological process. The second task is to sequence the operations at the stations. Note that both tasks will be solved simultaneously.

In terms of scheduling theory, we study a multi-objective, multi-stage hybrid shop scheduling problem with alternative machines at each stage with different time-dependent processing speeds, time-dependent machine availability, and ordered jobs, where job splitting is allowed. Examples of such problems can be found in the studies by Kyparisis & Koulamas (2006) and Tahar et al. (2006). The peculiarity of the problem under consideration is the simultaneous consideration of both the selection of the process design structure and the operation assignment. On the one hand, an assignment problem is discrete by nature and requires the introduction of binary variables, i.e., in this case discrete optimization techniques are appropriate. At the same time, the execution of a non-stationary operation can be accurately described in terms of continuous optimization. An additional peculiarity of such a simultaneous consideration is that both the machine structures and the flow parameters may be uncertain and changes in dynamics are therefore non-stationary.

According to the problem statement (see Fig. 1), we integrate and synchronize the following processes:

- Customer order fulfillment dynamics in regard to the process design and operation sequencing,
- Processing and transportation channel utilization dynamics,
- Material supply and consumption dynamics in the assembly system,
- Processing and movement dynamics in regard to processing and transportation channels in the assembly system.

3.2. Model

Let us introduce the following basic sets and structures (indices (o), (k), (r) and (f) describe the relations of the sets to the operations (o), channels (k), resources (r), and material flows (f) respectively).

$A = \{A_\nu, \nu \in 1,...n\}$ is the set of customer orders (jobs).

$M = \{M_i, i \in 1,...m\}$ is the set of stations in the assembly line.

$\tilde{B} = A \cup M$ is the union of the assembly line stations and the customer orders, i.e., the customized assembly system.

$C = \left\{C_\lambda^{(i)}, \lambda \in 1,...l_i\right\}$ is the set of channels at the stations and in-between the stations i and j in the assembly line, where index j is used for stations that receive products from an i-station.

$D = \left\{\{D_{æ'}^{(i)}\} \cup \{D_{æ}^{(i,j)}\}, æ, æ' \in 1,...s_i\right\}$ is the set of operations which can be executed in the assembly system.

$\Phi = \left\{\{\Phi S_\pi^{(i)}\} \cup \{\Phi N_\mu^{(i)}\}, \pi \in K_i^{(\mathrm{r},1)}, \mu \in K_i^{(\mathrm{r},2)}\right\}$ is the set of resources at the stations in the assembly line, where K is the set of numbers.

$\Phi S^{(i)} = \left\{\Phi S_\pi^{(i)}, \pi \in K_i^{(\mathrm{r},1)}\right\}$ is the set of storable resources at $M^{(i)}$ and $\Phi N^{(i)} = \left\{\Phi N_\mu^{(i)}, \mu \in K_i^{(\mathrm{r},2)}\right\}$ is the set of non-storable resources at $M^{(i)}$.

$P = \left\{\{P_{<æ',\rho>}^{(i)}\} \cup \{P_{<æ,\rho>}^{(i,j)}\}, \rho \in K_i^{(\mathrm{f})}\right\}$ is the set of material flows in the manufacturing process.

$P^{(i)} = \left\{P_{<æ',\rho>}^{(i)}\right\}$ is the set of material flows for the ρ-types of materials subject to $M^{(j)}$.

$P^{(i,j)} = \left\{P_{<æ,\rho>}^{(i,j)}\right\}$ is the set of material flows for the ρ-types of materials subject to $M^{(i)}$ and $M^{(j)}$.

The sets $\Gamma_{\nu 1}$, $\Gamma_{\nu 2}$ define "and" and "or" precedence relations for different jobs and the sets $\Gamma_{iæ1}, \Gamma_{iæ2}$ define "and" and "or" precedence relations for the operations $D_{æ}^{(i,j)}$ and $D_{æ'}^{(i)}$, respectively.

Assume that the manufacturing and transportation capacities may be disrupted and

- the availability of a station can be described by a given preset matrix time function $\varepsilon_{ij}(t)$ of time-spatial constraints: we have $\varepsilon_{ij}(t)=1$, if the station is available and $\varepsilon_{ij}(t)=0$, otherwise;
- the availability of a channel at a station can be described by the function $\Theta_{i\langle\varphi\lambda}(\tau)$ (or $\Theta_{\nu\langle\varphi\lambda}(\tau)$) which is equal to 1 if there are available channels, and equals 0, otherwise;
- the capacity degradation and recovery dynamics can be described by a continuous function of the perturbation impacts $\xi_{ij}(t)$; $\xi_{ij}(t)=1$ if the channel is 100% available and $\xi_{ij}(t)=0$ if the channel is fully disrupted. All other values for $\xi_{ij}(t)$ in the interval [0;1] are possible.

The formal statement of the scheduling problem is based on a dynamic interpretation of the execution processes of the operations. Let us introduce some new notations.

Parameters

$a_\alpha^{(o,1)}, a_\beta^{(o,1)}, a_{i\tilde{\alpha}}^{(o,2,\nu)}, a_{i\tilde{\beta}}^{(o,2,\nu)}, a_{is_i}^{(o,2,\nu)}$, $a_{iæ}^{(o,1,\nu)}$ are the planned manufacturing and transportation quantities for each operation. The values of these parameters are related to the end conditions that need to be reached in $x_\alpha^{(o,1)}(t), x_\beta^{(o,1)}(t)$ $x_{i\tilde{\alpha}}^{(o,2,\nu)}(t), x_{i\tilde{\beta}}^{(o,2,\nu)}(t)$, $x_{is_i}^{(o,2,\nu)}(t)$, $x_{iæ}^{(o,1,\nu)}(t)$ at $t = T_f$;

$\boldsymbol{h}_0^{(o)}, \boldsymbol{h}_1^{(o)}$ are known differential functions for the start and end conditions subject to the state variables $\boldsymbol{x}^{(o)}$ at $t = T_0$ and $t = T_f$.

$\mathbf{h}_0^{(k)}, \mathbf{h}_1^{(k)}$ are known differential functions for the start and end conditions in regard to the state vector $\boldsymbol{x}^{(\kappa)}$.

$b_{i'æ'iæ}^{(j,\lambda)}$ is the channel setup time.

$d_{iæj\lambda}^{(\pi)}$, $g_{iæj\lambda}^{(\mu)}$ are given consumption rates of the resources $\Phi S_{\pi}^{(j)}$ and $\Phi N_{\mu}^{(j)}$ for $D_{æ}^{(i,j)}$ and $C_{\lambda}^{(j)}$.

$\tilde{H}_j^{(\pi)}(t), \tilde{\tilde{H}}_j^{(\mu)}(t)$ are known rates for the replenishment of the resources $\Phi S_{\pi}^{(j)}$ and $\Phi N_{\mu}^{(j)}$, respectively.

$a_{j\lambda\pi(\eta-1)}^{(p,3)}, a_{j\lambda\mu(\eta'-1)}^{(p,4)}$ are known volumes (quantities) of the resource replenishment at the (η–1)th recovery cycle; $\tilde{\rho}_{\lambda}, \tilde{\tilde{\rho}}_{\lambda}$ are the numbers of the replenishment cycles.

$a_{iæ\rho}^{(f,1)}$ is the known lot size of a product type ρ for each operation $D_{æ}^{(i,j)}$.

$\tilde{P}_j^{(1)}$, $\tilde{P}_{j\rho}^{(2)}, \tilde{P}_{ij}^{(3)}$ are known values for the maximum storage capacities at M_j, handling (throughput) at M_j for ρ, and the transportation between M_i and M_j;

$c_{iæj\lambda\rho}^{(f,1)}$ is the maximum processing rate for the operation $D_{æ}^{(i,j)}$ at the λ-channel; it determines the maximum possible value for the production rate $u_{iæj\lambda\rho}^{(f,1)}$.

Decision variables

$x_{\nu}^{(0,1)}(t)$ is a state variable characterizing the lead time for job A_ν at each moment t;

$x_{iæ}^{(0,2,\nu)}(t)$ is a state variable characterizing the flow time of the operation $D_{æ}^{(i)}$ or $D_{æ}^{(i,j)}$);

$x_{\nu}^{(0,3)}(t)$ is a state variable characterizing the gap between the planned completion time for all jobs and the actual completion time of the job A_ν.

$u_{\nu j}^{(0,1)}(t), u_{iæj}^{(0,2,\nu)}(t), u_{\nu j}^{(0,3)}(t)$ are control variables; if $u_{\nu j}^{(0,1)}(t)=1$, then we have a transportation of job A_ν to B_j, and $u_{\nu j}^{(0,1)}(t)=0$ otherwise; $u_{iæj\lambda}^{(0,2,\nu)}(t)=1$ if operation $D_{æ}^{(i)}$ or $D_{æ}^{(i,j)}$ is assigned to a λ-channel, and $u_{iæj\lambda}^{(0,2,\nu)}(t)=0$ otherwise; $u_{\nu j}^{(0,3)}(t)=1$ at the moment when A_ν is completed at time point t until $t=T_f$, and $u_{\nu j}^{(0,3)}(t)=0$ otherwise.

$x_{iæj\lambda}^{(\kappa,1)}(t)$ is the state variable for the channel $C_{\lambda}^{(i)}$ at M_j during the setup to prepare the channel for processing $D_{æ'}^{(i,j)}$ after completing the operation $D_{æ}^{(i,j)}$.

$u_{iæj\lambda}^{(\kappa,1)}(t)$ is a control variable; $u_{iæj\lambda}^{(\kappa,1)}(t)=1$ if $C_{\lambda}^{(i)}$ is in the setup process and $u_{iæj\lambda}^{(\kappa,1)}(t)=0$ otherwise.

$x_{j\lambda}^{(\kappa,2)}(t)$ is a state variable characterizing the process (run) time of a channel.

$x_{j\lambda\pi}^{(\mathrm{p},1)}(t)$, $x_{j\lambda\mu}^{(\mathrm{p},2)}(t)$, $x_{j\lambda\pi\eta}^{(\mathrm{p},1)}(t)$, $x_{j\lambda\mu\eta'}^{(\mathrm{p},2)}(t)$ are state variables that characterize the current quantity (volume) of non-storable resources $\Phi S_{\pi}^{(j)}$, storable resources $\Phi N_{\mu}^{(j)}$, non-storable and recoverable (at stages η and η′) resources, and storable and recoverable (at stages η and η′) resources subject to channel $C_{\lambda}^{(j)}$, respectively. These state variables characterize a π-resource consumption and replenishment.

$x_{j\lambda\pi\eta}^{(\mathrm{p},3)}(t)$, $x_{j\lambda\mu\eta'}^{(\mathrm{p},4)}(t)$ are auxiliary state variables which are needed to define the sequence of the resource replenishments and the ends of the replenishment intervals, respectively.

$u_{j\lambda\pi\eta}^{(\mathrm{p},1)}, u_{j\lambda\mu\eta}^{(\mathrm{p},2)}$ are control variables characterizing the replenishment process for the non-storable and storable resources respectively; $u_{j\lambda\pi\eta}^{(\mathrm{p},1)}, u_{j\lambda\mu\eta}^{(\mathrm{p},2)}=1$ if a π-resource is under replenishment at time point *t*, and equal 0 otherwise.

$x_{iæj\lambda\rho}^{(\mathrm{f},1)}(t)$ is a state variable characterizing the quantity (volume) of the product «ρ» which is being delivered at M_j from M_i during the execution of $D_{æ}^{(i,j)}$ (or the processed quantity at M_j, if $i = j$);

$x_{iæj\lambda\rho}^{(\mathrm{f},2)}(t)$ is an auxiliary state variable characterizing the total processing time (including the waiting time) of a product flow ρ resulting from the interaction of M_i and M_j for $D_{æ}^{(i,j)}$ at $C_{\lambda}^{(i)}, C_{\lambda}^{(j)}$;

$u_{iæj\lambda\rho}^{(\mathrm{f},1)}$ is the shipment rate for the transportation from M_i to M_j (or the processing rate at M_j if $i = j$); $u_{iæj\lambda\rho}^{(\mathrm{f},2)}(t)$ is an auxiliary control variable; $u_{iæj\lambda\rho}^{(\mathrm{f},2)}(t)=1$ if processing at M_j is completed, and $u_{iæj\lambda\rho}^{(\mathrm{f},2)}(t)=0$ otherwise, or if, after the completion of $D_{æ}^{(i,j)}$ (or $D_{æ}^{(i)}$, if $i = j$), the next operation in the technological process $D_{\tilde{æ}}^{(i,j)}$ (or $D_{\tilde{æ}}^{(i)}$, if $i = j$) begins.

3.2.1 Process model of the operation execution

$$\dot{x}_{\nu}^{(0,1)} = \sum_{j=1}^{m} u_{\nu j}^{(0,1)};\ \dot{x}_{iæ}^{(0,2,\nu)} = \sum_{j=1}^{m}\sum_{\lambda=1}^{l_j} \varepsilon_{ij}(t)\Theta_{iæj\lambda}(t)u_{iæj\lambda}^{(0,2,\nu)};\ \dot{x}_{\nu j}^{(0,3)} = u_{\nu j}^{(0,3)}; \tag{8}$$

$$\dot{x}_{iæj\lambda}^{(\kappa,1)} = \sum_{j'=1}^{m}\sum_{æ'=1}^{s_i} \Theta_{i'æ'jæ} u_{i'æ'j\lambda}^{(\kappa,1)} \frac{b_{i'æ'iæ}^{(j,\lambda)} - x_{iæj\lambda}^{(\kappa,1)}}{x_{i'æ'j\lambda}^{(\kappa,1)}}; \tag{9}$$

$$\dot{x}_{j\lambda}^{(\kappa,2)} = \sum_{i=1}^{m}\sum_{æ=1}^{s_i}\left(u_{iæj\lambda}^{(o,2)} + u_{iæj\lambda}^{(\kappa,1)}\right). \tag{10}$$

$$\dot{x}_{j\lambda\pi}^{(\mathrm{p},1)} = -\sum_{i=1}^{m}\sum_{æ=1}^{s_i} d_{iæj\lambda}^{(\pi)}\left(u_{iæj\lambda}^{(\mathrm{o},2)} + u_{iæj\lambda}^{(\kappa,1)}\right); \tag{11}$$

$$\dot{x}_{j\lambda\mu}^{(\mathrm{p},2)} = -\sum_{i=1}^{m}\sum_{æ=1}^{s_i} g_{iæj\lambda}^{(\mu)}\left(u_{iæj\lambda}^{(\mathrm{o},2)} + u_{iæj\lambda}^{(\kappa,1)}\right); \tag{12}$$

$$x_{j\lambda\pi\eta}^{(\mathrm{p},1)} = -\sum_{i=1}^{m}\sum_{æ=1}^{s_i} d_{iæj\lambda}^{(\pi)}\left(u_{iæj\lambda}^{(\mathrm{o},2)} + u_{iæj\lambda}^{(\kappa,1)}\right) + u_{j\lambda\pi(\eta-1)}^{(\mathrm{p},1)}\,; \tag{13}$$

$$x_{j\lambda\mu\eta'}^{(\mathrm{p},2)} = -\sum_{i=1}^{m}\sum_{æ=1}^{s_i} g_{iæj\lambda}^{(\mu)}\left(u_{iæj\lambda}^{(\mathrm{o},2)} + u_{iæj\lambda}^{(\kappa,1)}\right) + u_{j\lambda\mu(\eta'-1)}^{(\mathrm{p},2)}\,; \tag{14}$$

$$x_{j\lambda\pi\eta}^{(\mathrm{p},3)} = u_{j\lambda\pi\eta}^{(\mathrm{p},1)};\ \ x_{j\lambda\mu\eta'}^{(\mathrm{p},4)} = u_{j\lambda\mu\eta'}^{(\mathrm{p},2)}. \tag{15}$$

$$x_{iæj\lambda\rho}^{(\mathrm{f},1)} = u_{iæj\lambda\rho}^{(\mathrm{f},1)};\quad x_{iæj\lambda\rho}^{(\mathrm{f},2)} = u_{iæj\lambda\rho}^{(\mathrm{f},2)}; \tag{16}$$

Eq. (8) describes the dynamics of the operation execution for the job A_ν. If $x_\nu^{(0,1)} = \sum_{j=1}^{m} u_{\nu j}^{(0,1)}$, then at each time point where, if $u_{\nu j}^{(0,1)}(t)$ =1 (i.e., the volume of the state variable x_ν is increasing), the job processing is in progress at the j-station in the assembly system. If $x_{iæ}^{(0,2,\nu)} = \sum_{j=1}^{m} \varepsilon_{ij}(t) u_{iæj}^{(0,2,\nu)}$, then the operation processing can start subject to the time windows of feasible capacity. If $x_{\nu j}^{(0,3)} = u_{\nu j}^{(0,3)}$, then the job is completed earlier than the due date (i.e., the earliness of the job completion subject to the slack time).

Eqs. (9) and (10) describe the state dynamics of the channel $C_\lambda^{(i)}$ at M_i and characterize the availability of the channel for the processing of operation $D_æ^{(i,j)}$. Eq. (9) describes the setup dynamics, and Eq. (10) reflects the occupation time of each channel subject to the dynamics of the operation execution (i.e., variable $u_{iæj\lambda}^{(o,2)}$ =1) in the Eq. (8).

Eqs. (11)-(15) describe the resource consumption dynamics (Eqs. 11 and 12) and the resource replenishment dynamics (Eqs. 13-15) subject to the assignment and setup decisions in Eqs. (8)-(10). Finally, Eq. (16) describes the material flow dynamics in the assembly system subject to the operation assignments to the stations, setups, and resource management decisions.

3.2.2 Constraints

$$\sum_{j=1}^{m} u_{\nu j}^{(0,1)}\left[\sum_{\alpha\in\Gamma_{\nu 1}}\left(a_\alpha^{(0,1)} - x_\alpha^{(0,1)}(t)\right) + \prod_{\beta\in\Gamma_{\nu 2}}\left(a_\beta^{(0,1)} - x_\beta^{(0,1)}(t)\right)\right] = 0; \tag{17}$$

$$\sum_{\lambda=1}^{l_j} u_{iæj\lambda}^{(0,2,\nu)}\left[\sum_{\tilde\alpha\in\Gamma_{iæ1}}\left(a_{i\tilde\alpha}^{(0,2,\nu)} - x_{i\tilde\alpha}^{(0,2,\nu)}(t)\right) + \prod_{\tilde\beta\in\Gamma_{iæ2}}\left(a_{i\tilde\beta}^{(0,2,\nu)} - x_{i\tilde\beta}^{(0,2,\nu)}(t)\right)\right] = 0; \tag{18}$$

$$\sum_{\nu=1}^{u} u_{\nu j}^{(0,1)}(t) \le 1, \forall j;\ \sum_{j=1}^{m} u_{\nu j}^{(0,1)}(t) \le 1,\ \ \forall j;\ u_{\nu j}^{(0,1)}(t) \in \{0,1\}; \tag{19}$$

$$u_{iæj\lambda}^{(0,2,\nu)}(t) \in \left\{0, u_{\nu j}^{(0,1)}\right\};\ u_{\nu j}^{(0,3)}(t) \in \{0,1\};\ u_{\nu j}^{(0,3)}\left(a_{js_i}^{(0,2,\nu)} - x_{js_j}^{(0,2,\nu)}(t)\right) = 0. \tag{20}$$

$$u_{iæj\lambda}^{(0,2)} x_{iæj\lambda}^{(\kappa,1)} = 0;\ \ x_{iæj\lambda}^{(\kappa,1)}(t) \in \{0,1\}; \tag{21}$$

$$\sum_{i=1}^{n}\sum_{æ=1}^{s_i} u_{iæj\lambda}^{(\kappa,1)}(t) \le 1,\ \ \forall j, \forall\lambda. \tag{22}$$

$$\sum_{i,æ,\lambda} d^{(\pi)}_{iæj\lambda}\left(u^{(o,2)}_{iæj\lambda} + u^{(\kappa,1)}_{iæj\lambda}\right) \le \tilde{H}^{(\pi)}_{j}(t); \tag{23}$$

$$\sum_{i,æ,\lambda} \int_{T_0}^{T_f} g^{(\mu)}_{iæj\lambda}\left(u^{(o,2)}_{iæj\lambda}(\tau) + u^{(\kappa,1)}_{iæj\lambda}(\tau)\right) d\tau \le \int_{T_0}^{T_f} \tilde{\tilde{H}}^{(\mu)}_{j}(\tau)d\tau; \tag{24}$$

$$u^{(p,1)}_{j\lambda\pi\eta}\left(a^{(p,3)}_{j\lambda\pi(\eta-1)} - x^{(p,3)}_{j\lambda\pi(\eta-1)}\right) = 0, \quad u^{(p,1)}_{j\lambda\pi\eta} x^{(p,1)}_{j\lambda\pi\eta} = 0. \tag{25}$$

$$u^{(p,2)}_{j\lambda\mu\eta}\left(a^{(p,4)}_{j\lambda\pi(\eta'-1)} - x^{(p,4)}_{j\lambda\pi(\eta'-1)}\right) = 0, \quad u^{(p,2)}_{j\lambda\mu\eta'} x^{(p,2)}_{j\lambda\mu\eta'} = 0. \tag{26}$$

$$u^{(p,1)}_{j\lambda\pi\eta}(t), u^{(p,2)}_{j\lambda\mu\eta}(t) \in \{0,1\}, \ \eta = 1,\ldots,\tilde{\rho}_{\lambda}; \ \eta' = 1,\ldots,\tilde{\tilde{\rho}}_{\lambda}. \tag{27}$$

$$0 \le u^{(f,1)}_{iæj\lambda\rho} \le c^{(f,1)}_{iæj\lambda\rho} u^{(o,2)}_{iæj\lambda}; \tag{28}$$

$$u^{(f,2)}_{iæj\lambda\rho}\left(a^{(f,1)}_{iæ\rho} - x^{(f,1)}_{iæj\lambda\rho}\right) = 0; \ u^{(f,2)}_{iæj\lambda\rho} x^{(o,2)}_{i\tilde{æ}} = 0; \ u^{(f,2)}_{iæj\lambda\rho}(t) \in \{0,1\}; \tag{29}$$

$$\sum_{i=1}^{m}\sum_{\lambda=1}^{l_i}\sum_{æ=1}^{s_i}\sum_{\rho=1}^{k_i} x^{(f,1)}_{iæj\lambda\rho}\left(u^{(o,2)}_{iæj\lambda\rho} + {}^{(f,2)}_{iæj\lambda\rho}\right) \le \tilde{P}^{(1)}_{j}; \tag{30}$$

$$\sum_{i=1}^{m}\sum_{\lambda=1}^{l_i}\sum_{æ=1}^{s_i} u^{(f,1)}_{iæj\lambda\rho} \le \tilde{P}^{(2)}_{j\rho}; \tag{31}$$

$$\sum_{\lambda=1}^{l_i}\sum_{æ=1}^{s_i}\sum_{\rho=1}^{k_i} u^{(f,1)}_{iæj\lambda\rho} \le \tilde{P}^{(3)}_{ij}. \tag{32}$$

Constraints (17) and (18) describe the technological precedence relations in regard to the jobs and operations of the jobs. Constraint (19) defines the rules of operation splitting and overlapping. Eq. (20) is a binary constraint on the control variables.

Constraints (21) and (22) determine the setup sequence at the channels and the conditions for setups at the channel $C^{(i)}_{\lambda}$. According to constraints (23) and (24), the intensities of the maximum resource consumption at each time point *t* are constrained subject to $\tilde{H}^{(\pi)}_{j}, \tilde{\tilde{H}}^{(\mu)}_{j}$. Constraints (25)-(27) determine the sequence of the replenishment actions. Eqs. (28)-(32) constrain the maximum processing rates subject to the operation assignments.

3.2.3. Boundary conditions

$$h^{(o)}_{0}\left(x^{(o)}(T_0)\right) \le 0; \quad h^{(o)}_{1}\left(x^{(o)}(T_f)\right) \le 0. \tag{33}$$

$$h^{(k)}_{0}\left(x^{(k)}(T_0)\right) \le 0; \quad h^{(k)}_{1}\left(x^{(k)}(T_f)\right) \le 0. \tag{34}$$

$$h^{(r)}_{0}\left(x^{(r)}(T_0)\right) \le 0; \quad h^{(r)}_{1}\left(x^{(r)}(T_f)\right) \le 0. \tag{35}$$

$$h^{(f)}_{0}\left(x^{(f)}(T_0)\right) \le 0; \quad h^{(f)}_{1}\left(x^{(f)}(T_f)\right) \le 0. \tag{36}$$

Eqs. (33)-(36) determine the initial and final values for the state variables in regard to the operations, channel, resource, and flow dynamics.

3.2.4. Control functionals

$$J_1^{(\mathrm{o})} = \sum_{\nu=1}^{n}\sum_{j=1}^{m} u_{\nu j}^{(o,3)}(T_f); \tag{37}$$

$$J_{<2,\alpha,\nu>}^{(\mathrm{o})} = \sum_{i=1}^{m}\sum_{j=1}^{m}\left(x_{\alpha i}^{(o,3)}(T_f) - x_{\nu j}^{(o,3)}(T_f)\right) \tag{38}$$

$$J_3^{(\mathrm{o})} = T_f - \sum_{j=1} x_{nj}^{(o,1)}(T_f) \tag{39}$$

$$J_{<4,i,\nu>}^{(\mathrm{o})} = \sum_{\nu,j,\lambda,æ} \int_{T_0}^{T_f} \varepsilon_{ij}(\tau)\Theta_{iæj\lambda}(\tau)u_{iæj\lambda}^{(0,2,\nu)}(\tau)d\tau, \tag{40}$$

$$J_{<5,i>}^{(\mathrm{o})} = \sum_{\nu,j,æ}\int_{T_0}^{T_f}\left[\varepsilon_{ij}(\tau) - \varepsilon_{ij}(\tau)u_{iæj\lambda}^{(0,2,\nu)}(\tau)\right]d\tau; \tag{41}$$

$$J_6^{(\mathrm{o})} = \sum_{i=1}^{m}\sum_{æ=1}^{s_i}\left(a_{iæ}^{(o,2,\nu)} - x_{iæ}^{(o,2,\nu)}(T_f)\right)^2; \tag{42}$$

$$J_7^{(\mathrm{o})} = \sum_{\nu=1}^{n}\sum_{i=1}^{m}\sum_{æ=1}^{s_i}\sum_{j=1}^{m}\sum_{\lambda=1}^{l_j}\int_{T_0}^{T_f}\tilde{\tilde{\beta}}_{iæ}^{(\nu)}(\tau)u_{iæj\lambda}^{(0,2,\nu)}(\tau)d\tau \tag{43}$$

$$J_1^{(\kappa)} = \sum_{\Delta_1=1}^{m-1}\sum_{\Delta_2=\Delta_1+1}^{m}\sum_{\lambda=1}^{l}\sum_{\zeta=1}^{l}\int_{T_0}^{T_f}\left(x_{\Delta_1\lambda}^{(\kappa,2)}(\tau) - x_{\Delta_2\zeta}^{(k,2)}(\tau)\right)d\tau; \tag{44}$$

$$J_2^{(\kappa)} = \sum_{\Delta_1=1}^{m-1}\sum_{\Delta_2=\Delta_1+1}^{m}\sum_{\lambda=1}^{l}\sum_{\zeta=1}^{l}\left(x_{\Delta_1\lambda}^{(\kappa,2)}(T_f) - x_{\Delta_2\zeta}^{(\kappa,2)}(T_f)\right), \tag{45}$$

$$J_{1j\pi}^{(\mathrm{p})} = \sum_{\lambda=1}^{l_j}\sum_{\eta=1}^{\tilde{\rho}_\lambda} x_{j\lambda\pi\eta}^{(\mathrm{p},3)}; \tag{46}$$

$$J_{2j\mu}^{(\mathrm{p})} = \sum_{\lambda=1}^{l_j}\sum_{\eta'=1}^{\tilde{\tilde{\rho}}_\lambda} x_{j\lambda\mu\eta}^{(\mathrm{p},4)}, \tag{47}$$

$$J_1^{(\mathrm{f})} = \sum_{i=1}^{m}\sum_{æ=1}^{s_i}\sum_{\substack{j=1\\ i\neq j}}^{m}\sum_{\lambda=1}^{l_i}\sum_{\rho=1}^{k_i}\left(a_{iæ\rho}^{(\mathrm{f},1)} - x_{iæj\lambda\rho}^{(\mathrm{f},1)}\right)^2\Bigg|_{t=T_f} \tag{48}$$

$$J_2^{(\mathrm{f})} = \sum_{i=1}^{m}\sum_{æ=1}^{s_i}\sum_{\substack{j=1\\ i\neq j}}^{m}\sum_{\lambda=1}^{l_i}\sum_{\rho=1}^{k_i}\int_{T_0}^{T_f} x_{iæj\lambda}^{(\mathrm{f},2)}(\tau)d\tau \tag{49}$$

We refer to the studies (Ivanov et al. 2010, Ivanov and Sokolov 2012) for a multi-objective resolution of optimal control scheduling models.

The control functional $J_1^{(\mathrm{o})}$ (Eq. 37) characterizes the overall number of completed jobs in the assembly system at $t = T_f$. This is the performance indicator for the assembly system *throughput*. $J_{<2,\alpha,\nu>}^{(\mathrm{o})}$ (Eq. 38) reflects the lead time for the job A_ν. $J_3^{(\mathrm{o})}$ (Eq. 39) characterizes the *makespan* for all jobs A_ν. $J_{<4,i,\nu>}^{(\mathrm{o})}$ (Eq. 40) characterizes the *processing time* of job A_ν. $J_{<5,i>}^{(\mathrm{o})}$

(Eq. 41) is the *waiting time* of job A_v. $J_6^{(o)}$ (Eq. 42) depicts the degree of the job completion at the end of the planning interval. $J_7^{(o)}$ (Eq. 43) expresses the *total tardiness* for all operations subject to the penalty functions $\tilde{\tilde{\beta}}_{iæ}^{(\nu)}$, i.e., an *on-time-delivery*.

The control functionals (44) and (45) estimate the equality of the channel utilization at the stations in the assembly system at each time point $t \in (T_0, T_f]$ and at the end of the planning interval. The control functionals (46) and (47) estimate the degree of the resource replenishment and the timeliness of the resource replenishment, respectively. The control functional (48) characterizes the gap between the planned and actually processed operation volume and is interconnected with the control functional (42). The control functional (49) depicts the waiting time for the operations and is interconnected with the control functional (41).

3.2.5. Integration principle

To obtain a constructive solution to the problem considered, we propose to use a functorial transition from the category of digraphs (*Cat*Φ) that specifies the manufacturing technology to the category of dynamic models (*Cat*D), which describes the operation execution. The covariant functor G: Φ→D sets the state relations in-between the nodes in the manufacturing technology plan and the operation execution schedule. The simplified mathematical model of manufacturing technology plan and the operation execution schedule integration can be presented as shown in Eq. (50):

$$\begin{aligned}\Delta = \Bigg\{ & \mathbf{u} \,|\, \frac{dx_i}{dt} = \sum_{\nu=1}^{n} u_{\nu j};\ \sum_{j=1}^{m} u_{\nu j} \le 1; \sum_{\nu=1}^{n} u_{\nu j} \le 1; u_{\nu j}(t) \in \{0,1\}; \\ & \sum_{\nu=1}^{n} u_{\nu j} \left[\sum_{\alpha \in \Gamma_1^-} (a_\alpha - x_\alpha(t)) + \prod_{\beta \in \Gamma_2^-} (a_\beta - x_\beta(t)) \right] = 0; \\ & t \in (T_0, T_f] = T;\ x_\nu(T_0) = 0; x_\nu(T_f) = a_\nu \Bigg\}\end{aligned} \tag{50}$$

where x_v is a variable characterizing the state of the job A_v, $u_{vj} = 1$ is a control action ($u_{vj} = 1$, if the station $M^{(i)}$ is used for job $A^{(\nu)}$), a_v, a_α, a_β are given quantities (end conditions), the values of which should have the corresponding variables $x_v(t)$, $x_\alpha(t)$, $x_\beta(t)$ at the end of the planning interval at the time point $t = T_f$, t is the running time point, T_0 is the start time point of the planning horizon, T_f is the end time point of the planning horizon, T is the planning horizon, $\sum_{\nu=1}^{n} u_{vj} \sum_{a \in \Gamma_1^-} (a_\alpha - x_\alpha(t)) = 0$ are constraints "and" which relate the condition of the total processing of all predecessor operations,

$\sum_{\nu=1}^{n} u_{vj} \prod_{\beta \in \Gamma_2^-} (a_\beta - x_\beta(t)) = 0$ are constraints "or" which relate the condition of the processing of at least one of the predecessor operations, and Γ_1^-, Γ_2^- are the sets of processes which immediately precede the job $A^{(\nu)}$.

In the proposed approach, the assignment and flow control are considered simultaneously. Since the task times may differ subject to a varying processing speed $c_{iæj\lambda\rho}^{(f,1)}$ and the channel availability $\varepsilon_{ij}(t)$ and $\Theta_{iæj\lambda}(t)$, the assignments made on the basis of the planned processing volumes $a_{\nu j}$ are forwarded to the resource and flow dynamics control models and further optimized in regard to resource consumption, replenishment, and usage over time. In the flow control model, the assignment of an operation to a channel and the execution start of the operation at the channel cause dynamic flows of the processed products.

3.2.5 Formulation of the scheduling problem

The task is to find a feasible control $\mathbf{u}(t)$, $[T_0, T_f)$ which ensures that the dynamic control model meets the constraint functions and guides the dynamic system (i.e., the schedule) $\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}, \mathbf{u})$ from the initial state to the specified final state subject to given end conditions and the uncertainty area under the disturbances $\boldsymbol{\xi}(t)$. If there are several feasible controls (schedules), then the best one (optimal) should be selected in order to maximize (minimize) the control functionals (37)-(49).

3.3 Extension towards the Industry 4.0 schedule control

Industry 4.0 scheduling implies both the process design and operation sequencing. As such, scheduling problems with *hybrid structural-terminal-logical constraints* must be analyzed (Dolgui et al. 2018). Consider the simplified example of a system with *hybrid structural-terminal-logical constraints* in Fig. 3.

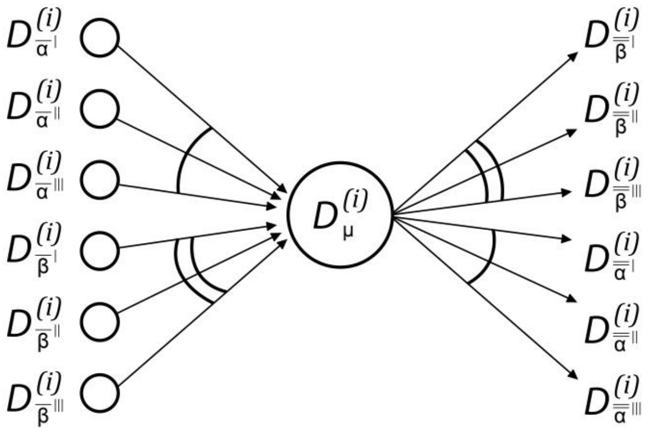

Fig. 3. Precedence relations of operation $D_{\mu}^{(i)}$

Let us illustrate a general logical-dynamic mathematical model of the production process structure and the process control. This model implicitly enables the description of all possible interconnections of the production structures and processes on the basis of the operation execution control. In order to differentiate this model from the general case described in Sect. 4, we change several notations. In particular, the operation index is now μ instead of æ; the job index is now i instead of ν.

In Fig. 3, the operation $D_{\mu}^{(i)}$ follows the operations $D_{\bar{\alpha}'}^{(i)}$ $D_{\bar{\alpha}''}^{(i)}$ $D_{\bar{\alpha}'''}^{(i)}$ according to the "and" precedence rule and the operations $D_{\bar{\beta}'}^{(i)}$ $D_{\bar{\beta}''}^{(i)}$ $D_{\bar{\beta}'''}^{(i)}$ according to the precedence rule "or". Analogously, the six operations follow the operation $D_{\mu}^{(i)}$ according to either "and" or "or" rules. For the system considered, the following modified optimal control model can be presented (Eqs. 51-53).

$$\dot{x}_{i\mu}^{(o)} = \sum_{j=1}^{m} \varepsilon_{ij}(t)\Theta_{i\mu j}(t)(u_{i\mu j}^{(o)}(t) + w_{i\mu j}^{(o)}(t)) \tag{51}$$

$$\dot{x}_{i\mu}^{(f)} = \sum_{j=1}^{m} u_{i\mu j}^{(f)}(t) \tag{52}$$

$$\dot{x}_{j}^{(k)} = \sum_{i=1}^{n}\sum_{\mu=1}^{s_i} u_{i\mu j}^{(o)} \tag{53}$$

The interpretation of this model is similar to the model (8)-(16).

Eqs. (54)-(56) describe the requirements on the non-preemption of the operations and the numerical assessment of the number of completed operations as well as on the determination of the operation completion time, respectively (Kalinin and Sokolov 1985).

$$\dot{z}_{i\mu j} = u_{i\mu j}^{(o)} + w_{i\mu j}^{(o)};\ \dot{h}_{i\mu j} = z_{i\mu j};\ \dot{g}_{i\mu j} = \eta_{i\mu j} \tag{54}$$

$$\dot{x}_{i\mu}^{(b,1)} = u_{i\mu}^{(b,1)} \tag{55}$$

$$\dot{x}_{i\mu}^{(b,1)} = u_{i\mu}^{(b,2)} \tag{56}$$

The constraint system (17)-(32) can now be modified as shown in Eqs. (57)-(70).

$$\sum_{j=1}^{m} u_{i\mu j}^{(f)}(t) \le 1; \forall i, \forall \mu;\quad \sum_{i=1}^{f}\sum_{\mu=1}^{s_i} u_{i\mu j}^{(o)}(t) \le 1\cdot \xi_j^{(1)}; \forall j \tag{57}$$

$$\sum_{j=1}^{m} u_{i\bar{\alpha} j}^{(o)} \cdot \sum_{\mu\in\Gamma_{i\bar{\alpha}}^{+}} x_{i\mu}^{(o)} = 0 \tag{58}$$

$$\sum_{j=1}^{m} u_{i\bar{\beta} j}^{(o)} \cdot \prod_{\mu\in\Gamma_{i\bar{\alpha}}^{+}} x_{i\mu}^{(o)} = 0 \tag{59}$$

$$\sum_{j=1}^{m} u_{i\bar{\bar{\alpha}} j}^{(o)} \cdot \sum_{\mu\in\Gamma_{i\bar{\bar{\alpha}}}^{-}} (a_{i\mu}^{(f)} - x_{i\mu}^{(f)}) = 0 \tag{60}$$

$$\sum_{j=1}^{m} u_{i\bar{\bar{\beta}} j}^{(o)} \cdot \prod_{\mu\in\Gamma_{i\bar{\bar{\beta}}}^{-}} (a_{i\mu}^{(f)} - x_{i\mu}^{(o)}) = 0 \tag{61}$$

$$\sum_{j=1}^{m} w_{i\mu j}^{(o)}[a_{i\mu}^{(f)} - x_{i\mu}^{(f)}] = 0 \tag{62}$$

$$w_{i\mu j}^{(o)} \cdot g_{i\mu j} = 0 \tag{63}$$

$$x_{i\mu j}[a_{i\mu}^{(o)} - \sum_{j=1}^{m} z_{i\mu j}] = 0 \tag{64}$$

$$\left[\sum_{j=1}^{m} (z_{i\mu j} \cdot g_{i\mu j} + \frac{(a_{i\mu})^2}{2} - h_{i\mu j})^2 \cdot z_{i\mu j}^2 \right] \Bigg|_{t=T_f} = 0 \tag{65}$$

$$u_{i\mu}^{(b,1)} \cdot x_{i\mu}^{(b,2)} = 0 \tag{66}$$

$$u_{i\mu}^{(b,2)}[a_{i\mu}^{(f)} - x_{i\mu}^{(f)}] = 0 \tag{67}$$

$$0 \le \xi_j^{(1)}(t) \le 1 \tag{68}$$

$$0 \le \xi_j^{(2)}(t) \le 1 \tag{69}$$

$$u_{i\mu j}^{(o)}(t),\ w_{i\mu j}^{(o)}(t),\ \eta_{i\mu j}(t),\ u_{i\mu}^{(b,1)}(t),\ u_{i\mu}^{(b,2)}(t) \in \{0,1\}, \tag{70}$$

where $z_{ij\mu}^{(o)}$ is an auxiliary variable which characterizes the execution of the μ-operation, $h_{ij\mu}^{(o)}$ is the square under the integral curve $z_{ij\mu}^{(o)}$, and $g_{ij\mu}^{(o)}$ is an auxiliary variable which is equal to the time point $t_{ij\mu}^{/}$ between the completion time of the μ-operation and T_f.

Constraints (58) and (59) define precedence relations for operation $D_{\mu}^{(i)}$ with regard to the predecessor operations $D_{\bar{\alpha}}^{(i)}$, $D_{\bar{\beta}}^{(i)}$. Constraints (60) and (61) define precedence relations for operation $D_{\mu}^{(i)}$ with regard to the subsequent operations $D_{\bar{\bar{\alpha}}}^{(i)}$, $D_{\bar{\bar{\beta}}}^{(i)}$. Constraint (62) defines the logic for the auxiliary control variable $w_{i\mu j}^{(o)} \in \{0,1\}$ which equals 1 if $x_{i\mu}^{(f)}(t) = a_{i\mu}^{(f)}$ at time point t and $x_{i\mu}^{(o)} \ne a_{i\mu}^{(o)}$. In other words, the flow is interrupted. To compensate this, an auxiliary control $w_{i\mu j}^{(o)}$ is introduced in Eq. (51) which differentiates it from Eq. (8). Constraint (63) is used to avoid overproduction regarding the operation $D_{\mu}^{(i)}$, i.e., $x_{i\mu}^{(o)}(T_f) = a_{i\mu}^{(o)}$, which means that $x_{i\mu}^{(o)}(T_f)$ cannot exceed $a_{i\mu}^{(o)}$. Constraints (65) are used jointly with constraints (63) and (64) to avoid interruptions of an operation execution (Kalinin and Sokolov 1987, Ivanov et al. 2013). With the help of constraints (66) and (67), the jobs can be counted at $t = T_f$ for which all the required operations $D_{\mu}^{(i)}$ are completed. In order to assess the schedule robustness (e.g., using attainable sets), we introduce constraints on perturbations (68) and (69).

The boundary conditions can be written as shown in Eqs. (71) and (72).

$$\begin{gathered} t = T_o:\ x_{i\mu}^{(o)}(T_o) = x_{i\mu}^{(f)}(T_o) = x_j^{(k)}(T_o) = z_{i\mu j}(T_o) = u_{i\mu j}(T_o) = g_{i\mu j}(T_o) = 0 \\ x_{i\mu}^{(b,1)}(T_o) = 0;\quad x_{i\mu}^{(b,2)}(T_o) = 0 \end{gathered} \tag{71}$$

$$\begin{gathered} t = T_f:\ x_{i\mu}^{(o)}(T_f) = a_{i\mu}^{(o)};\quad x_{i\mu}^{(f)}(T_f) = a_{i\mu}^{(f)} \\ x_i^{(f)}(T_f),\ z_{i\mu j}(T_f),\ h_{i\mu j}(T_f),\ g_{i\mu j}(T_f),\ x_{i\mu}^{(b,1)}(T_f),\ x_{i\mu}^{(b,2)}(T_f) \in \Re^1,\ \Re^1 \in [0,...,\infty) \end{gathered} \tag{72}$$

The control functionals are introduced in Eqs. (73)-(76).

$$J_1^{add} = \frac{1}{2}\sum_{i=1}^{n}\sum_{\mu=1}^{s_i}\left[(a_{i\mu}^{(o)} - \sum_{j=1}^{m} z_{i\mu j})^2 + \sum_{j=1}^{m}(z_{i\mu j}g_{i\mu j} + \frac{(a_{i\mu})^2}{2} - h_{i\mu j})^2 z_{i\mu j}^2\right]_{t=T_f} \tag{73}$$

$$J_2^{add} = \sum_{i=1}^{n}\frac{x_{is}^{(b,2)}(T_f)}{T_f - x_{is}^{(b,1)}(T_f)} \tag{74}$$

$$J_3^{(add)} = \sum_{\mu=1}^{s_i}\frac{x_{i\mu}^{(b,2)}(T_f)}{T_f - x_{i\mu}^{(b,1)}(T_f)} \tag{75}$$

$$J_4^{(add)} = (J_1^{(\min)} - J_1^{(\xi)})(J_2^{(\min)} - J_2^{(\xi)}) \tag{76}$$

Eq. (73) considers a non-preemptive execution of an operation. Eq. (74) counts the number of completed jobs at $t = T_f$. Eq. (75) counts the number of completed operations $D_\mu^{(i)}$ at $t = T_f$ (Sokolov and Yusupov 2002). Eq. (76) is used to estimate the schedule robustness using attainable sets (Ivanov and Sokolov 2010, Ivanov et al. 2016a,b) which enables to determine the existence of a feasible scheduling solution subject to the feasibility analysis of a two-point boundary problem.

4. Algorithm

4.1. Computational principles

The major algorithmic idea proposed in this research is to decompose the assignment matrix dynamically in time, considering polynomially solvable (by tendency) problems of small dimensionality at each time point (Fig. 4).

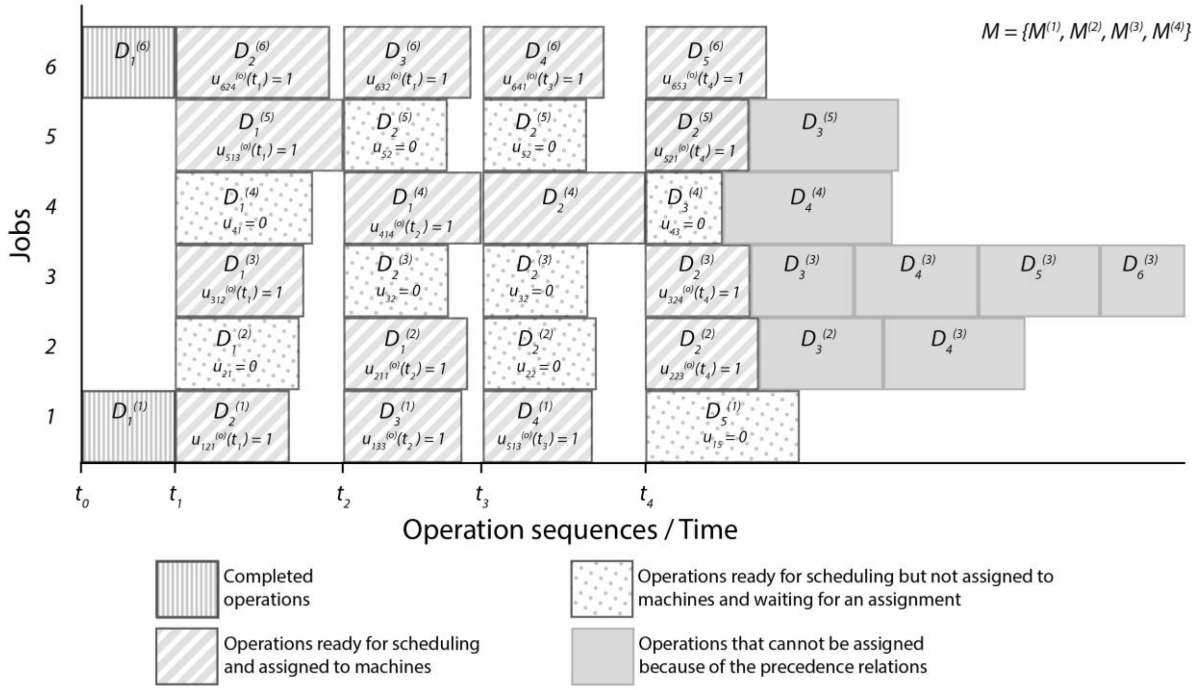

Fig. 4. Dynamic decomposition principle

In the example illustrated in Fig. 4, six jobs are considered each of which contains a sequence of operations that can be executed at four alternative workstations. At time point $t=t_1$, the operations D_{62}, D_{51}, D_{41}, D_{31}, D_{21}, and D_{12} are available for an assignment to machines subject to precedence relations. Therefore, a small-dimensional assignment problem with six operations and four machines arises at time point $t=t_1$ which is solved using standard discrete optimization techniques towards Hamiltonian extremization. As a result, the operations D_{62}, D_{51}, D_{31}, and D_{12} are assigned to the machines $M^{(4)}$, $M^{(3)}$, $M^{(2)}$, $M^{(1)}$, respectively. This is indicated by the control variables u_{624}, u_{513}, u_{312}, and u_{121}, which are switched to 1. Similarly, at time point $t=t_2$, we deal with a small-dimensional assignment problem with six operations and four machines. As a result, the operations D_{63}, D_{41}, D_{22}, and D_{13} are assigned to the machines $M^{(2)}$, $M^{(4)}$, $M^{(1)}$, $M^{(3)}$, respectively.

To summarize and generalize the example in Fig. 4, the small dimensionality at each time point results from the dynamic decomposition described by the constraints on the precedence relations, i.e., at each time point we consider only those operations that can be assigned to machines at that time point, excluding those operations that have already been completed and those that cannot start because the predecessors are not completed yet. By dynamically decomposing the assignment matrix in time, we represent the optimal control problem as a two-point boundary problem, then to treat polynomially solvable problems (by tendency) of small dimensionality at each time point, and then to integrate these partial solutions through the maximum principle by integrating the main and adjoint equation systems.

It is commonly known that analytical methods for an optimal control have been proven for small-dimensional systems. In the control engineering practice, numerical methods have been applied. A methodical challenge in applying the maximum principle is to find the coefficients of the adjoint system which change in dynamics. Another methodical challenge of boundary problems is that the initial conditions for the adjoint variables $\psi(T_0)$ are not given. At the same time, an optimal program control should be calculated subject to the end conditions.

4.2. Computational procedure

The following algorithm has been developed to compute an optimal schedule in an Industry 4.0 production system. The computational procedure for the developed model (see the main system (51) – (53)) is based on the integration of the main and adjoint equation systems subject to the maximization of the following Hamiltonian (77)-(79):

$$H(\mathbf{x}^*(t),\mathbf{u}^*(t),\psi(t)^*)=\max_{\tilde{\tilde{\mathbf{u}}}\in\tilde{\tilde{\mathbf{Q}}}(\mathbf{x})}\sum_{z=1}^{2}H_l(\mathbf{x}(t),\mathbf{u}(t),\psi(t)), \tag{77}$$

$$H_1=\sum_{i=1}^{\bar{n}}\sum_{\mu=1}^{s_i}\sum_{j=1}^{n}[\psi_{i\mu}^{(o)}\cdot\varepsilon_{ij}+\psi_j^{(k)}+\lambda_2^{(o)}\alpha_{i\mu j}^{(o)}]u_{i\mu j}^{(o)}, \tag{78}$$

$$H_2=\sum_{i=1}^{\bar{n}}\sum_{\mu=1}^{s_i}\sum_{j=1}^{n}[\psi_{i\mu j}^{(f)}+\lambda_5^{(f)}\beta_{i\mu}^{(f)}]u_{i\mu j}^{(f)}, \tag{79}$$

where $\boldsymbol{\psi}(t)$ is the adjoint vector.

The maximization of the Hamiltonian H_1 for model (30) in combination with the constraints (34) solves the assignment problem. The maximization of the Hamiltonian H_2 for model (32) in combination with the constraints (34)-(35) solves the linear programming problem. At each time point, only operations and constraints which meet the requirement are considered (33). By a dynamic switching of the constraints (33) from inequalities to equalities, the size of the scheduling problem at each time point is reduced. The Hamiltonians (78) and (79) can be maximized when the constraints (33) satisfy the corresponding variables $u_{i\mu j}^{(o)}$ and $u_{i\mu j}^{(f)}$. In this case, only a part of the constraints (33) is considered for the current assignment problem since, when the control in (33) is switched to zero, it becomes active in the right-hand part of the equations (34). Therefore, the reduction of the problem dimensionality at each time point in the calculation process is ensured because of the recurrent operation description.

In accordance with the maximum principle, the following conjugate system (corresponding to the main system (51)-(53)) can be written (80)-(82):

$$\frac{d\psi_{i\mu}^{(o)}}{dt} = \dot{\psi}_{i\mu}^{(o)} = -\sum_{j=1}^{n}[\psi_{i(\mu+1)}^{(o)}\varepsilon_{ij} + \psi_j^{(k)} + \lambda_2^{(o)}\alpha_{i(\mu+1)j}^{(o)}]u_{i(\mu+1)j}^{(o)} , \quad (80)$$

$$\frac{d\psi_j^{(k)}}{dt} = \dot{\psi}_j^{(k)} = 0 , \quad (81)$$

$$\frac{d\psi_{i\mu j}^{(f)}}{dt} = \dot{\psi}_{i\mu j}^{(f)} = 0 . \quad (82)$$

The transversality conditions can be formulated in the following way (83)-(85):

$$\psi_{i\mu}^{(o)}(T_f) = \lambda_1^{(o)}(a_{i\mu}^{(o)} - x_{i\mu}^{(o)}(T_f)); \ (83)$$

$$\psi_j^{(k)}(T_f) = \lambda_3^{(k)}(T - x_j^{(k)}(T_f)); \quad (84)$$

$$\psi_{i\mu j}^{(f)}(T_f) = \lambda_5^{(f)}(a_{i\mu j}^{(f)} - x_{i\mu j}^{(f)}(T_f)); (85)$$

More specifically, for scheduling an Industry 4.0 system, the following Hamiltonian can be written (Eqs. 86-92):

$\mathrm{H}(\vec{x}(t), \vec{u}(t), \vec{\psi}(t)) = \mathrm{H}_1 + \mathrm{H}_2 + \mathrm{H}_3 + \mathrm{H}_4 + \mathrm{H}_5 + \mathrm{H}_6 + \mathrm{H}_7$, where

$$\mathrm{H}_1 = \sum_i \sum_j \sum_\mu [\psi_{i\mu}^{(o)}\varepsilon_{ij}\Theta_{i\mu j} + \psi_j^{(k)} + \varphi_{i\mu j}^{(1)} + \alpha_{i\mu} - \beta_{i\mu}] \cdot u_{i\mu j}^{(o)} , \quad (86)$$

$$\mathrm{H}_2 = \sum_i \sum_\mu \sum_j [\psi_{i\mu}^{(f)}] \cdot u_{i\mu j}^{(f)} \quad (87)$$

$$\mathrm{H}_3 = \sum_i \sum_\mu \sum_j [\varphi_{i\mu j}^{(1)} + \psi_{i\mu}^{(o)} \cdot \varepsilon_{ij}\Theta_{i\mu j}] \cdot w_{i\mu j}^{(o)} \quad (88)$$

$$\mathrm{H}_4 = \sum_i \sum_\mu \sum_j \varphi_{i\mu j}^{(2)} \cdot z_{i\mu j} \quad (89)$$

$$\mathrm{H}_5 = \sum_i \sum_\mu \sum_j \varphi_{i\mu j}^{(3)} \cdot \eta_{i\mu j} \quad (90)$$

$$\mathrm{H}_6 = \sum_i \sum_\mu p_{i\mu}^{(b,1)} \cdot u_{i\mu}^{(b,1)} \tag{91}$$

$$\mathrm{H}_7 = \sum_i \sum_\mu p_{i\mu}^{(b,2)} \cdot u_{i\mu}^{(b,2)} \tag{92}$$

Define the adjoint equation system (Eqs. 93-99) $\vec{\psi} = \left\| \psi_{i\mu}^{(o)} \psi_{i\mu}^{(f)} \psi_j^{(k)} \varphi_{i\mu j}^{(1)} \varphi_{i\mu j}^{(2)} \varphi_{i\mu j}^{(3)} p_{i\mu}^{(b,1)} p_{i\mu}^{(b,2)} \right\|^{\mathrm{T}}$

according to the main equation system (51)-(56) $\vec{x} = \left\| x_{i\mu}^{(o)} x_{i\mu}^{(f)} x_j^{(k)} z_{i\mu j} h_{i\mu j} g_{i\mu j} x_{i\mu}^{(b,1)} x_{i\mu}^{(b,2)} \right\|^{\mathrm{T}}$

$$\dot{\psi}_{i\mu}^{(o)} = -\sum_{\bar{\alpha} \in \Gamma_{i\mu}^-} \sigma_{i\bar{\alpha}}^{(and)} \cdot \sum_{j=1}^m u_{i\bar{\alpha}j}^{(o)} - \sum_{\bar{\beta} \in \Gamma_{i\mu}^-} \sigma_{i\bar{\beta}}^{(or)} \cdot \sum_{j=1}^m u_{i\bar{\beta}j}^{(o)} \cdot \prod_{\substack{\delta \in \Gamma_{i\beta}^+ \\ \delta \neq \mu}} x_{i\delta}^{(o)}\ ; \tag{93}$$

$$\dot{\psi}_{i\mu}^{(f)} = -\sum_{\bar{\bar{\alpha}} \in \Gamma_{i\mu}^+} \sigma_{i\bar{\bar{\alpha}}}^{(and)} \cdot \sum_{j=1}^m u_{i\mu j}^{(o)} - \sum_{\bar{\bar{\beta}} \in \Gamma_{i\mu}^+} \sigma_{i\bar{\bar{\beta}}}^{(or)} \cdot \sum_{j=1}^m u_{i\bar{\bar{\beta}}j}^{(o)} \cdot \prod_{\substack{\gamma \in \Gamma_{i\beta}^- \\ \gamma \neq \mu}} (a_{i\gamma}^{(f)} - x_{i\gamma}^{(f)}) - \sigma_{i\mu}^{(2,1)} \cdot \sum_{j=1}^u w_{i\mu j}^{(o)} - \sigma_{i\mu}^{(5,2)} \cdot u_{i\mu}^{(b,2)}\ ; \tag{94}$$

$$\dot{\psi}_o^{(k)} = 0 \tag{95}$$

$$\dot{\varphi}_{i\mu j}^{(1)} = -\varphi_{i\mu j}^{(2)} - \sigma_{i\mu j}^{(4,1)} \eta_{i\mu j} \tag{96}$$

$$\dot{\varphi}_{i\mu j}^{(2)} = 0 \tag{97}$$

$$\dot{\psi}_{i\mu j}^{(3)} = -\sigma_{i\mu}^{(2,1)} \cdot w_{i\mu j}^{(o)} \tag{98}$$

$$\dot{p}_{i\mu}^{(b,1)} = 0 \tag{99}$$

$$\dot{p}_{i\mu}^{(6b,2)} = 0 \tag{100}$$

where $\sigma_{i\bar{\alpha}}^{(and)}$; $\sigma_{i\bar{\beta}}^{(or)}$; $\sigma_{i\bar{\bar{\beta}}}^{(or)}$; $\sigma_{i\bar{\bar{\alpha}}}^{(and)}$; $\sigma_{i\mu}^{(5,2)}$; $\sigma_{i\mu}^{(2,1)}$ are the computed coefficients that are similar to the Lagrange multipliers in mathematical programming, but which change dynamically. The analytical form of the computation of the coefficients is presented in Ivanov et al. (2016a).

The transversality conditions for the case form Sect. 3.3 can be written as Eqs. (101)-(108):

$$\psi_{i\mu}^{(o)}(T_f) = -(a_{i\mu}^{(o)} - x_{i\mu}^{(o)})\Big|_{t=T_f} \tag{101}$$

$$\psi_{i\mu}^{(f)}(T_f) = (a_{i\mu}^{(f)} - x_{i\mu}^{(f)})\Big|_{t=T_f} \tag{102}$$

$$\psi_j^{(k)}(T_f) = (\tilde{t}_f - x_j^{(k)}(T_f)) \tag{103}$$

$$\phi_{i\mu j}^{(1)}(T_f) = (a_{i\mu}^{(o)} - \sum_{j=1}^m z_{i\mu j})\Big|_{t=T_f} + \left[(z_{i\mu j} g_{i\mu j} + \frac{(a_{i\mu})^2}{2} - h_{i\mu j}) z_{i\mu j}{}^2 g_{i\mu j} + (z_{i\mu j} g_{i\mu j} + \frac{(a_{i\mu})^2}{2} - h_{i\mu j})^2 z_{i\mu j} \right]\Bigg|_{t=T_f} \tag{104}$$

$$\phi_{i\mu j}^{(2)}(T_f) = z_{i\mu j}{}^2 (z_{i\mu j} g_{i\mu j} + \frac{(a_{i\mu})^2}{2} - h_{i\mu j})\Big|_{t=T_f} \quad (105)$$

$$\phi_{i\mu j}^{(3)}(T_f) = -z_{i\mu j}{}^2 (z_{i\mu j} g_{i\mu j} + \frac{(a_{i\mu})^2}{2} - h_{i\mu j})\Big|_{t=T_f} \quad (106)$$

$$p_{i\mu}^{(b,1)}(T_f) = + \frac{x_{is}^{(b,1)}(T_f)}{[T_f - x_{is}^{(b,1)}(T_f)]^2}\Big|_{t=T_f} \quad (107)$$

$$p_{i\mu}^{(b,2)}(T_f) = - \frac{1}{[T_f - x_{i\mu}^{(b,2)}]}\Big|_{t=T_f} \quad (108)$$

After the transformation into a boundary problem, a relaxed problem can be solved to receive an optimal program control, for the computation of which the main and adjoint systems are integrated, i.e., the optimal program control vector $\mathbf{u}^*(t)$ and the state trajectory $\mathbf{x}^*(t)$ are obtained. The optimal program control vector at time $t = T_0$ for the given value of $\boldsymbol{\psi}(t)$ should return the maximum to (77)-(79), which have been transformed to a general performance index and expressed in scalar form J_G.

The basic peculiarity of the boundary problem considered is that the initial conditions for the adjoint variables $\psi(T_0)$ are not given. At the same time, an optimal program control should be calculated subject to the boundary conditions (we omit their presentation and refer to the paper (Ivanov et al. 2016a, Sokolov et al. 2018). To obtain the adjoint system vector, the Krylov–Chernousko method for an optimal program control problem with a free right end has been used combined with the modified successive approximation (MSA) method.

Step 1 An initial solution $\bar{\mathbf{u}}(t)$, $t \in (T_0, T_f]$ (a feasible control, in other words, a feasible schedule) is selected and $r = 0$.

Step 2 As a result of the dynamic model run, $\mathbf{x}^{(r)}(t)$ is generated. In addition, if $t = T_f$ then the record value $J_G = J_G^{(r)}$ can be calculated. After this, the transversality conditions (83)-(85) are evaluated. The dynamic coordination parameters are the adjoint variables which change their values during the iterative process of the solution of the corresponding two-point boundary problem. At each time point, the Hamiltonian function is extremized. The locally coordinated sub-problems are partial combinatorial assignment problems and LP problems which are formed dynamically depending on the precedence relations, the channel availability at the stations, and the operation execution progress.

Step 3 The adjoint system (93)-(96) [(80)-(82)] is integrated subject to $\mathbf{u}(t) = \bar{\mathbf{u}}(t)$ and over the interval from $t = T_f$ to $t = T_0$. For the time $t = T_0$, the first approximation $\boldsymbol{\psi}_i^{(r)}(T_0)$ is obtained as a result. Here, the iteration with the number $r = 0$ is completed. The values of the adjoint variables change from iteration to iteration. The coordination of the adjoint variables and the Hamiltonian functions allow the simultaneous coordination of the solutions for both the goal and the resource at each time point, which are found in the combinatorial sub-problems.

Step 4 From the time point $t = T_0$ onwards, the control $\mathbf{u}^{(r+1)}(t)$ is determined ($r = 0,1,2,...$ denotes the number of the iteration). In parallel with the maximization of the Hamiltonian, the main system of equations and the conjugate one are integrated. The maximization involves the solution of several mathematical programming problems at each time point.

In the aforementioned algorithm, the search for an optimal control in each iteration is performed in the class of boundary (e.g., pointwise or relay) functions which correspond to the discrete nature of decision making in scheduling. A methodical challenge in applying the maximum principle is to find the coefficients of the adjoint system which change in dynamics. For the given form of the control model, these coefficients can be found analytically according to the formulas presented in (Ivanov et al. 2016a). The formulated model is a linear, non-stationary, finite–dimensional controlled system of differential equations with a convex area of feasible control. This model form satisfies the conditions of the existence theorem in Lee and Markus (1967, Theorem 4, Corollary 2), which allows us to assert the existence of an optimal solution in the appropriate class of feasible controls. According to Lee and Markus (1967, Theorem 4, Corollary 2), along with the initial class $\tilde{K}$ formed via the constraints $\mathbf{q}^{(1)}$ and $\mathbf{q}^{(2)}$ describing the domain $\mathbf{Q}(\mathbf{x}(t))$, an extended class $\tilde{\tilde{K}}$ of control inputs can be considered. In the extended class $\tilde{\tilde{K}}$, the relay constraints $u_{i\mu j}^{(o)}(t) \in \{0;1\}$ are substituted by a less strict one: $u_{i\mu j}^{(o)}(t) \in [0;1]$ ($\mathbf{u}$ is substituted by $\tilde{\tilde{\mathbf{u}}}$). In this case, an extended domain $\tilde{\tilde{\mathbf{Q}}}(\mathbf{x}(t))$ of feasible control inputs may be formed by means of special transformations, ensuring the convexity and the compactness of $\mathbf{Q}(\mathbf{x}(t))$ (Moiseev, 1974). The aforementioned theorem of Lee and Markus (1967) confirms that all the conditions for the existence of an optimal control for the extended control class $\tilde{\tilde{K}}$ are valid. If in a given class of feasible control actions $\tilde{\tilde{K}}$, an optimal control $\tilde{\tilde{\mathbf{u}}}(t)$ exists, then, as it arises from the local cut method, the control $\tilde{\tilde{\mathbf{u}}}(t)$ returns at each time point $t \in (T_0, T_f]$ at the set $\tilde{\tilde{\mathbf{Q}}}(\mathbf{x}(t))$ the maximum of the Hamiltonian. Since $\tilde{\tilde{\mathbf{Q}}}(\mathbf{x}(t))$ is a linear capsule of $\mathbf{Q}(\mathbf{x}(t))$, the maximization of the Hamiltonian over the sets $\mathbf{Q}$ and $\tilde{\tilde{\mathbf{Q}}}$ leads to the same result. An optimal control for the class $\tilde{K}$ belongs to the class $\tilde{\tilde{K}}$. Taking into account $\tilde{K} \subset \tilde{\tilde{K}}$, this control is also optimal for the class $\tilde{K}$. Therefore, the relaxed problem can be solved instead of the initial one to obtain an optimal feasible control for the class $\tilde{K}$. An analysis of the studies (Boltyanskiy, 1973; Moiseev, 1974) shows that for a linear, non-stationary, finite-dimensional system with a convex area of feasible control $Q(\boldsymbol{x})$ and the goal vector, the stated necessary optimality conditions are also sufficient conditions.

Finally, the problem of operation preemptions needs to be addressed while applying the algorithm described above. Since the operations have different processing durations, the machines become available for the processing of the next operations at different time points. This can result in a situation, where the maximization of the Hamiltonian can lead to an improvement of the performance objective (e.g., job lateness or makespan) if the operation processing at one machine is interrupted and the operation is reassigned to the new machine that has become available. Practically speaking, this can arise due to the technology. In other settings, such preemptions can be prohibited. The study by Ivanov et al. (2013) illustrated how to resolve this problem at the modeling and algorithmic levels. The monotony and convergence of the modified successive approximations method for problems with a non-preemption condition has been previously proven for two-point linear boundary problems with convex control areas and goal functions (Lybushin 1979, Kalinin and Sokolov 1987).

For optimality and sufficiency proofs, a computational complexity analysis, a computational procedure with the help of maximizing the Hamiltonian, an analytical derivation of the transversality condition coefficients, and the algorithm for the computation of an optimal control, we refer to the study (Ivanov et al. 2016a) that shows that the developed modification of the MSA method guarantees a monotonic change of the adjoint variable values by both the transversality conditions and a situational selection of the function values of the Hamiltonian. The transversality conditions interconnect the state parameter values in the main and adjoint systems at the time point that corresponds to the end of the planning interval. The values of the Hamiltonian function are saved during the iterative search procedure. The theoretical convergence of the iterative procedure considered was previously proven in Lyubushin (1979) and Chernousko and Lyubushin (1982).

4.3. Use of Industry 4.0 and digital technology in the implementation of the dynamic schedule computation

The control algorithm proposed can be enriched by the integration of Industry 4.0 elements such as sensors and data analytics which may have two impacts on the system. The first is the tuning: changing uncertain coefficients in the structure of the differential equations of the system, taking into account that a larger number of these coefficients implies a more accurate system response to a changing environment. The second is the learning: imposing new restrictions on the system behavior. The number of arbitrary coefficients in the structure of the differential equations changes in the process of learning, imposing dynamic restriction adjustments on the behavior of the system.

The possibility to use real-time data of the machine availability and the operation processing status on the basis of digital technologies allows the realization of different dynamic decomposition principles in the optimal control algorithm. The selection of the time points at which the Hamiltonian is extremized and small-dimensional assignment problems are solved can be implemented not only as a fixed t-step procedure but also in the event-oriented form subject to such events as "a machine becomes available" or "a new job enters the system".

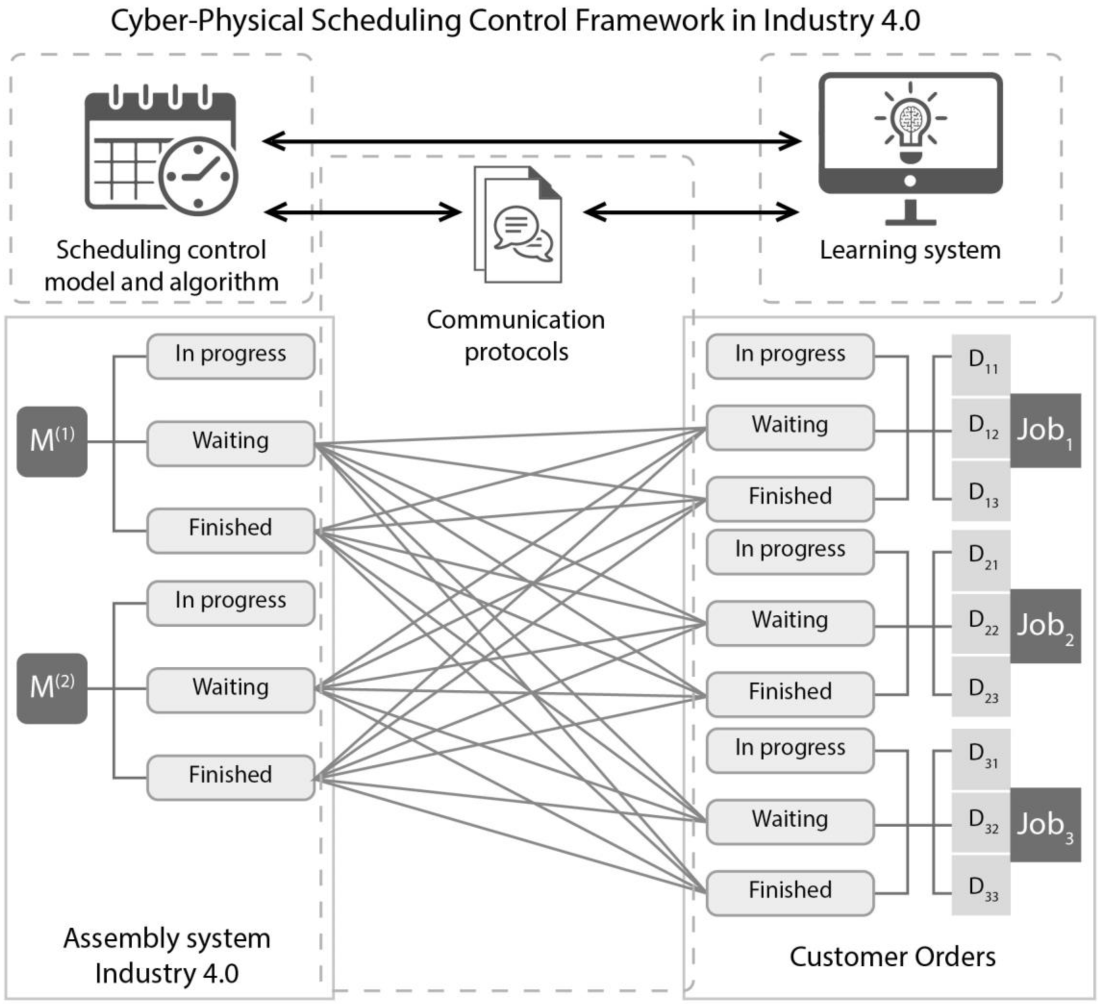

Fig. 5. Cyber-Physical Scheduling Control Framework in Industry 4.0

Consider the example illustrated in Fig. 5 with two machines M_1 and M_2 and three products. The manufacturing of the products is organized in three jobs each of which is characterized by the sequence of operations D_1-D_2-D_3 which can be processed on both M_1 and M_2. Both machines and products are equipped with sensors and a communication protocol is established between the sensors. The sensors observe the machine utilization and operation processing subject to three states, i.e., "in process", "waiting" and "finished". The scheduling algorithm is activated in the case of the following events in the communication protocol: (i) a machine signals the completion of an operation processing and there is at least one operation either in the state "waiting" or "finished", (ii) a product signals the completion of the processing of an operation on a machine and there is at least one machine either in the state "waiting" or "finished", and (iii) there are at least one machine and one operation in the state "waiting".

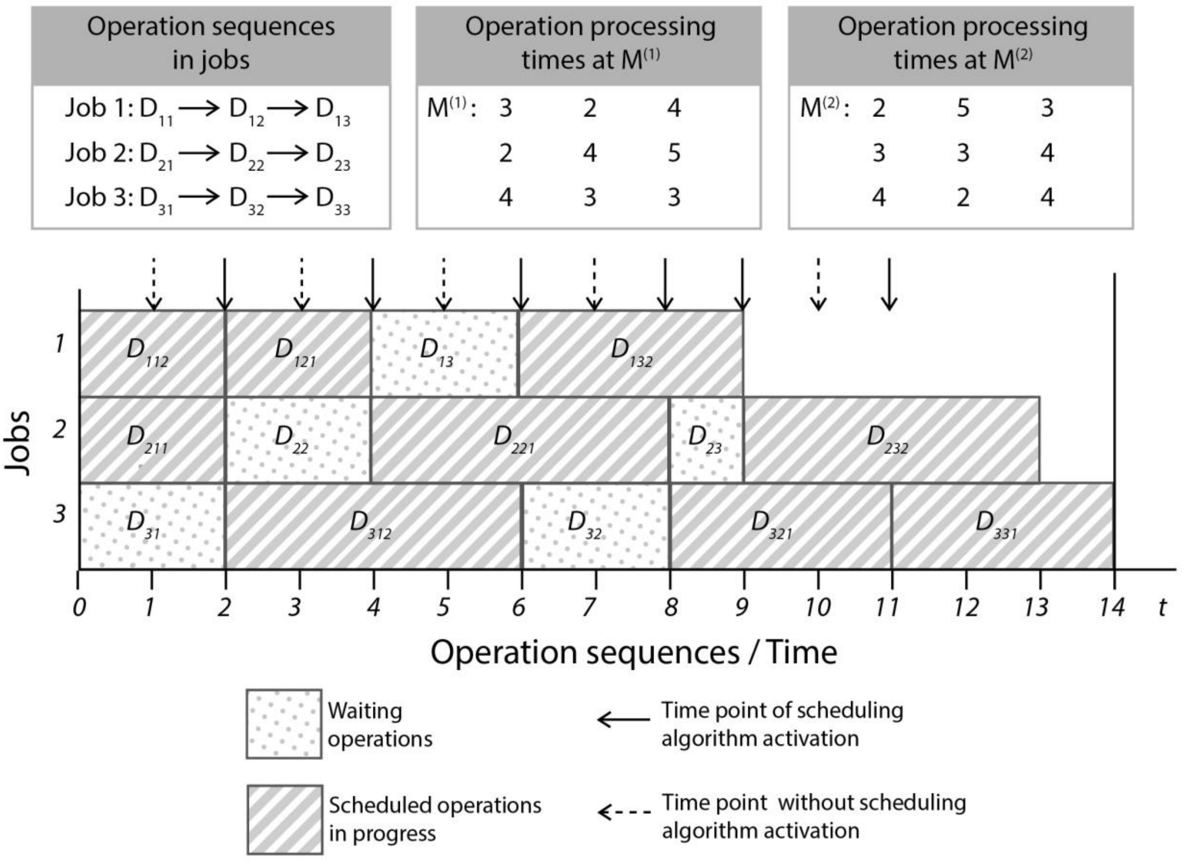

Fig. 6. Numerical example

Consider the numerical example given in Fig. 6, where the operation processing times of the operations arranged in three jobs on two alternative flexible machines are depicted. At time $t=T_0=0$, the operations D_{11}, D_{21}, and D_{31} can be processed and have therefore the state "waiting". Both machines M_1 and M_2 are available and have therefore the status "waiting". As such, the scheduling algorithm is activated and a discrete optimization algorithm is applied towards the maximization of the Hamiltonian for a problem with the three operations D_{11}, D_{21}, and D_{31} and the two machines M_1 and M_2 subject to some criteria from the set (37)-(49) represented in a generalized scalar form J_G. For simplification, Fig. 6 shows the optimization with regards to a single criterion (44) "equal machine utilization" at each point of time.

At time $t=T_1=1$, the operations D_{11} and D_{21} have the status "in process", D_{31} is in the state "waiting", and both machines M_1 and M_2 are occupied and have therefore the state "in process". At time $t=T_2=2$, the operations D_{11} and D_{21} signal the completion of the processing and change their states from "in process" to "finished" which automatically change the states of the operations D_{12} and D_{22} to "waiting". Operation D_{31} is still in the state "waiting", and both machines M_1 and M_2 change their states from "in process" to "finished". As such, the scheduling algorithm is activated and a discrete optimization algorithm is applied towards the maximization of the Hamiltonian for a problem with the three operations D_{12}, D_{22}, and D_{31} and the two machines M_1 and M_2. We note that in general case the optimization is running towards a scalar performance criterion J_G that results from scalarization of the criteria vector (37)-(49), cf. Step 2 of the scheduling algorithm and comments after Eq. (108). For details of the scalar transformation, we refer to Ivanov and Sokolov (2010).

Finally, we note that the model and algorithm developed can be applied both at the proactive and the reactive scheduling stages. At the proactive stage, they can be used for a descriptive and diagnostic analysis and a predictive modeling to analyze the possible performances of the production system. At the reactive stage, they can be used for the control of the real-time schedule and adaptive learning.

5. Conclusions

The Industry 4.0 technology enables new production strategies that require highly customized assembly systems. The ultimate objective of these systems is to facilitate a flexible customized manufacturing at the lower costs of mass production. Such innovative production strategies create a number of new challenges and opportunities for short-term job scheduling. In particular, the manufacturing processes for different customer orders may have individual machine structures such that the flexible stations are able to execute different functions subject to individual sets of operations within the jobs. Therefore, the problem of a simultaneous structural-functional synthesis of the customized assembly system arises. A flexible distributed scheduling as required by the Industry 4.0 paradigm has been addressed in this study using optimal program control theory.

The major contribution of this paper is the development of an optimal control model for the simultaneous structural-functional design of a customized manufacturing process and sequencing of the operations of the jobs in an Industry 4.0 system. For the first time, a multi-objective, multi-stage job shop scheduling problem with alternative and flexible machines at each stage and different time-dependent processing speeds and time-dependent machine availability, without job splitting was solved by means of optimal control and the maximum principle using the maximization of the Hamiltonian.

The method and the algorithm developed present a contribution to flexible, distributed scheduling in the emerging field of Industry 4.0-based innovative production systems. In contrast to previous studies which assumed a fixed process design, our approach is capable of designing simultaneously the manufacturing process in regard to the available alternative stations, their current capacity utilization and the processing time, and the sequencing of the jobs at the stations.

The basic *computational idea* of the computational approach developed is that the operation execution and machine availability are dynamically distributed in time over the planning horizon. As such, not all operations and machines are relevant to decision making at the same time. Therefore, the solution at each time point for a small-dimensional system is calculated by mathematical programming. The multi-dimensionality and the combinatorial explosion of the problem faces a decreasing connectivity under the network diagram of the operations. The analysis of the manufacturing process paths for the execution of different jobs can help to reveal a real utilization of the channels at different stations as well as the stations all together. This may be helpful for estimating the requirements on the multi-functionality of the stations. Such an analysis may reveal, e.g. that some channels are utilized fully while other channels are used occasionally. This analysis may be used for the capacity design and investment decisions.

The formulation of the scheduling model in the dynamic control form makes it possible to apply it both to proactive and reactive real-time scheduling. As such, integration of the planning and real-time control stages can be realized using the unified methodical and technical principles. Moreover, the formulation of the scheduling model as an optimal program control allows the consideration of a non-stationary process view and the use of the accuracy of continuous time. In addition, a wide range of analysis tools from control theory regarding stability, controllability, adaptability, etc. may be used if a schedule is described in terms of control.

In the future, a robustness analysis of the overall system, i.e., both of the process design and the schedule, can extend the results of this study. In addition, computational examples may help to reveal new insights. A more detailed analysis of the Industry 4.0 technology may illuminate a taxonomy of structural-functional problems in this emerging research field. Finally, Industry 4.0 and digital technologies open new possibilities to the implementation of dynamic scheduling techniques using real-time data about the machine utilization, entering of new jobs and the operation processing status. This makes it possible to extend the algorithmic dynamic decomposition principles of the control scheduling models, e.g., by incorporating an event-oriented decomposition based on such events as “a machine becomes available”, “an operation is completed” or “a new job enters the system”. These extensions can be considered in light of future research topics.

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Appendix 1 Notations

Sets, maps, constants

Notation	**Meaning**
$A = \{A_\nu, \nu \in 1,\ldots n\}$	Set of jobs
$D = \left\{ \begin{array}{l} \{D_{æ'}^{(i)}\} \cup \{D_{æ}^{(i,j)}\}, \\ i, j \in 1,..m, æ', æ \in 1,...s_i \end{array} \right\}$	Set of operations
$M = \{M_i, i \in 1,\ldots m\}$	Set of stations
$C = \{C_\lambda^{(i)}, \lambda \in 1, l_i\}$	Set of channels
$\Phi = \left\{ \begin{array}{l} \{\Phi S_\pi^{(i)}\} \cup \{\Phi N_\mu^{(i)}\}, \\ \pi \in K_i^{(\mathrm{r},1)}, \mu \in K_i^{(\mathrm{r},2)} \end{array} \right\}$	Set of resources
$\Phi S^{(i)} = \left\{ \Phi S_\pi^{(i)}, \pi \in K_i^{(\mathrm{r},1)} \right\}$	Set of storable resources at $M^{(i)}$
$\Phi N^{(i)} = \left\{ \Phi N_\mu^{(i)}, \mu \in K_i^{(\mathrm{r},2)} \right\}$	Set of non-storable resources at $M^{(i)}$;
K	Set of numbers
$P = \left\{ \{P_{<æ',\rho>}^{(i)}\} \cup \{P_{<æ,\rho>}^{(i,j)}\}, \rho \in K_i^{(\mathrm{f})} \right\}$	Set of material flows subject to $M^{(i)}$
$P^{(i,j)} = \left\{ P_{<æ,\rho>}^{(i,j)}, \rho \in K_i^{(\mathrm{f})} \right\}$	Set of material flows for the ρ-types of materials subject to $M^{(i)}$ and $M^{(j)}$.
$\Gamma_{\nu 1}, \Gamma_{\nu 2}$	Sets of “and” and “or” precedence relations for the jobs
$\Gamma_{iæ1}, \Gamma_{iæ2}$	Sets of “and” and “or” precedence relations for the operations
U	Set of feasible control inputs
J	Set of performance indicators
$\Pi_{<\delta,\delta'>}^{t}$	Map describing the allowable transitions from one multi-structural macro-state to another one
Δ	Set of dynamic and static alternatives of the manufacturing process
$R_{\tilde{r}}$	Set of process constraints
$\tilde{\tilde{R}}_{\tilde{r}}$	Constants, which are known and $T = (T_0, T_f]$ is the time interval for the manufacturing process design synthesis
$\zeta \in \{1,\ldots,\Im\}$	Set of the numbers of the performance indicator

$\mathbf{X}(\xi(t),t)$	Area of the allowable states of the assembly line structural dynamics

Parameters and functions

Notation	**Meaning**
a	Planned processing volume (lot size)
$\tilde{P}_j^{(1)}$, $\tilde{P}_{j\rho}^{(2)}$, $\tilde{P}_{ij}^{(3)}$	Known values for the maximal storage capacity at M_j, handling capacity (throughput) at M_j for ρ, and transportation capacity between M_i and M_j, respectively
T_0	Start time of the scheduling horizon
T_f	End time of the scheduling horizon
b	Setup time of a channel
$d_{iæj\lambda}^{(\pi)}$, $g_{iæj\lambda}^{(\mu)}$	Given consumption intensities of $\Phi S_{\pi}^{(j)}$ and $\Phi N_{\mu}^{(j)}$ for $D_{æ}^{(i,j)}$ and $C_{\lambda}^{(j)}$
$\tilde{H}_j^{(\pi)}(t)$, $\tilde{\tilde{H}}_j^{(\mu)}(t)$	Intensities for the replenishment of the resources of $\Phi S_{\pi}^{(j)}$ and $\Phi N_{\mu}^{(j)}$, respectively
$\xi(t)$	Vector of perturbation impacts
$\mathbf{h}_0^{(o)}$, $\mathbf{h}_1^{(o)}$	Differentiable functions that determine the end conditions of the vector
σ	Duration of the planning interval
$\varepsilon(t)$	Preset matrix time function of the time-spatial constraints for the stations
$\Theta_{iæj\lambda}(t)$	Preset matrix time function of the time-spatial constraints for the channels
$\tilde{\beta}(\tau)$	Penalty function for the completion delay of an operation
$\mathbf{q}^{(1)}$ and $\mathbf{q}^{(2)}$	Vector-functions, defining the main spatio-temporal, economic, technical and technological conditions for the machine functioning process.

Indices

Notation	**Meaning**
v	Job index
n	Running numbers of a job
æ	Operation index

s	Running numbers of an operation
i	Station index Job index from Sect. 3.6.2 ongoing
m	Running numbers of a station
λ	Channel index
l	Running numbers of a channel
r	Number of the iteration of the algorithm
ρ	Product flow index
π	Storable resource index
μ	Nonstorable resource index Operation index from Sect. 3.6.2 ongoing
η	Replenishment cycle
$\tilde{\rho}, \tilde{\tilde{\rho}}$	Running numbers of the replenishment cycles
α,β	Indices of the precedence relations “and”, “or” for the jobs
$\tilde{\alpha}, \tilde{\beta}$	Indices of the precedence relations “and”, “or” for the operations
l'	Running numbers of a structure element of the manufacturing process
δ	Running number of a multi-structural macro-state of the manufacturing process -
χ	Running number of the design structure of an alternative manufacturing process
(o), (k), (r), (f)	Indexes to describe the relationships of the respective sets to the operations (o), channels (k), machines (r), and material flows (f).
t	Current time point

Decision control and state variables

Notation	Meaning
$\mathbf{u}(t)$	Control variable
$\mathbf{x}(t)$	State variables
$z^{(o)}_{ij\mu}$	Auxiliary variable which characterizes the execution

	of the μ-operation
$h_{ij\mu}^{(o)}$	The square under the integral curve $z_{ij\mu}^{(o)}$,
$g_{ij\mu}^{(o)}$	Auxiliary variable which is equal to the time $t_{ij\mu}^{/}$ between the completion time of the μ-operation and T_f.
$w_{i\mu j}^{(o)}$	Auxiliary control variable which equals 1 if $x_{i\mu}^{(f)}(t) = a_{i\mu}^{(f)}$ at time t and $x_{i\mu}^{(o)} \neq a_{i\mu}^{(o)}$.
$\sigma_{i\overline{\alpha}}^{(and)}$; $\sigma_{i\overline{\beta}}^{(or)}$; $\sigma_{i\overline{\overline{\beta}}}^{(or)}$; $\sigma_{i\overline{\overline{\alpha}}}^{(and)}$; $\sigma_{i\mu}^{(5,2)}$; $\sigma_{i\mu}^{(2,1)}$	Coefficients of the adjoint system
$\psi(t)$	Adjoint variable