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Exercises "Numerical Methods in Fluid Mechanics"
Summer 2020 - 1

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1. An infinite long river

$$\Omega = \{(x,y) \in \mathbb{R}^2 \,:\, 0 < y < 1\}$$

is in motion. The deformation field $\hat{\mathbf{u}}(\hat{\mathbf{x}},t)$ for a particle $\hat{\mathbf{x}} = (\hat{x},\hat{y})$ is given as

$$\hat{\mathbf{u}}(\hat{\mathbf{x}},t) = \begin{pmatrix} \hat{y}(1-\hat{y}) \\ 0 \end{pmatrix} t.$$

a) Show that the velocity in Lagrangian coordinates and Eulerian coordinates is given by

$$\hat{\mathbf{v}}(\hat{\mathbf{x}},t) = \begin{pmatrix} \hat{y}(1-\hat{y}) \\ 0 \end{pmatrix}, \quad \mathbf{v}(\mathbf{x},t) = \begin{pmatrix} y(1-y) \\ 0 \end{pmatrix}.$$

Why is there no difference between both viewpoints here?

b) Assume that two ducks are floating in the river with initial position $\hat{\mathbf{x}} = \mathbf{x}_1(0) = (0, 0.2)$ and $\hat{\mathbf{x}}_2 = \mathbf{x}_2(0) = (0, 0.2+\delta)$ for a small $\delta > 0$. Compute the Deformation gradient as seen from duck $\mathbf{x}_1$. What does it say about the relation of the two ducks?

c) Compute the strain rate tensor, again for duck $\mathbf{x}_1$. What does it say about the relation of the two ducks?

2. Proof the following (simpler) one dimensional equivalent to Reynolds transport theorem. Do an elementary proof and do to transfer it to the general case:
Let $I = (a(t), b(t))$ be an interval given by two functions $a, b \in C^1(\mathbb{R})$ with $a(t) < b(t)$. It holds

$$\frac{\partial}{\partial t}\int_{a(t)}^{b(t)} f(x,t)\,dx = \int_{a(t)}^{b(t)} \frac{\partial}{\partial t} f(x,t) + \frac{\partial}{\partial x}\big(v(x,t) f(x,t)\big)\,dx,$$

where $v(x,t)$ describes the velocity of the interval motion.
Hint: One possible way to proof this result is a transformation of $(a(t), b(t))$ to $(0,1)$ via $x(\hat{x},t) = a(t) + \hat{x}(b(t) - a(t))$. This deformation function belongs to the velocity $\hat{v}(\hat{x},t) = a'(t) + \hat{x}(b'(t) - a'(t))$.

Remark You are not required to hand in this problem set. But, if returned (per mail) until April 24, I will correct the answers.