Determination of pressure data in aortic valves

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Motivation

Current medical methods in stenosis evaluation and pressure determination

Pressure determination - continuum mechanics approach

Pressure difference and energy dissipation dependence on the stenosis severity
AORTIC VALVE
Aortic valve


Motivation

Current medical methods in stenosis evaluation and pressure determination

Pressure determination - continuum mechanics approach

Pressure difference and energy dissipation dependence on the stenosis severity
Current methods in stenosis evaluation

- anatomic stenosis

  \[
  \text{severity} = \left( 1 - \frac{\text{area}_{\text{stenotic}}}{\text{area}_{\text{healthy}}} \right) \cdot 100\%
  \]

- physiologically important stenosis
  - valve area/effective orifice area
  - additional heart work/energy dissipation
  - trans-stenosis pressure difference
A cardiac cycle

Figure: Blood pressure in the left ventricle during a cardiac cycle. (from Wikipedia, modified.)

Phase 1, diastolic filling  
Phase 2, isovolumic contraction  
Phase 3, systolic ejection  
Phase 4, isovolumic relaxation
**Pressure difference**

\[
\frac{1}{2} \rho \ast v_1^2 + h_1 \rho \ast g \ast = \frac{1}{2} \rho \ast v_2^2 + h_2 \rho \ast g
\]

\[
(h_1 - h_2) \rho \ast g \ast = \frac{1}{2} \rho \ast v_2^2
\]

\[
(h_1 - h_2) = C v_2^2
\]

---

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Pressure difference and energy dissipation dependence on the stenosis severity
Pressure Poisson equation

\[ \nabla p = -\rho_* \left( (\nabla \mathbf{v}) \mathbf{v} - \frac{\partial \mathbf{v}}{\partial t} \right) + \text{div}(2\mu_* \mathbf{D}) =: \mathbf{f} \]

\[ -\Delta q_{ppe} = \text{div} \mathbf{f} \quad \text{in } \Omega \]

\[ \frac{\partial q_{ppe}}{\partial \mathbf{n}} = \mathbf{n} \cdot \mathbf{f} \quad \text{on } \partial \Omega \]

\[ q_{ppe} = p_* \quad \text{on } \Gamma_{out} \]
**Stokes equation**

\[
\nabla p = -\rho \left( (\nabla v) v - \frac{\partial v}{\partial t} \right) + \text{div}(2\mu D) =: f
\]

\[-\Delta a + \nabla q_{\text{ste}} = f \quad \text{in } \Omega,\]
\[
\text{div } a = 0 \quad \text{in } \Omega,\]
\[
a = 0 \quad \text{on } \partial \Omega,\]
\[
q_{\text{ste}} = p_* \quad \text{on } \Gamma_{out}.\]

**Pressure determination**

**INPUT data/4D MRI**

- time resolved phase-contrast magnetic resonance imaging 4D-PCMR (4D Flow MRI)
- limitations in both spatial and temporal resolution of the signals


- fixing pressure (catheterization)
Reference flow

incompressible unstationary Navier-Stokes equation, no-slip wall, velocity prescribed on the inlet with parabolic profile, pressure and the backflow stabilization/penalization prescribed on the outlet
**Comparison on the same mesh**

\[
\|q_{ppe} - p_{ref}\|_{L^2} \bigg/ \|p_{ref}\|_{L^2} \quad \|q_{ste} - p_{ref}\|_{L^2} \bigg/ \|p_{ref}\|_{L^2}
\]

<table>
<thead>
<tr>
<th></th>
<th>|q_{ppe} - p_{ref}|<em>{L^2} \bigg/ |p</em>{ref}|_{L^2}</th>
<th>|q_{ste} - p_{ref}|<em>{L^2} \bigg/ |p</em>{ref}|_{L^2}</th>
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<tbody>
<tr>
<td>symmetric</td>
<td>6.40e-04</td>
<td>1.50e-14</td>
</tr>
<tr>
<td>non-symmetric</td>
<td>3.50e-03</td>
<td>1.16e-14</td>
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</tbody>
</table>

**Table:** Relative errors for fine data.

**L0 mesh for symmetric case**

**L0 mesh for non-symmetric case**
Comparison on the coarser meshes

L4 mesh for symmetric case

L4 mesh for non-symmetric case

Pressure determination

$\text{Pressure determination}$

$\text{Comparison on the coarser meshes}$

$L0$ $L1$ $L2$ $L3$ $L4$

$L0$ $L1$ $L2$ $L3$ $L4$

$L4$ mesh for symmetric case

$L4$ mesh for non-symmetric case

Pressure [mmHg]

$\text{pressure [mmHg]}$

$z$ coordinate [mm]

$z$ coordinate [mm]

$q_{PPE}$

$q_{STE}$

$p_{REF}$

$q_{PPE}$

$q_{STE}$

$p_{REF}$

$16/37$
# Comparison on the Coarser Meshes

<table>
<thead>
<tr>
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<th>symmetric case</th>
<th>non-symmetric case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$err_{ppe}$</td>
<td>$err_{ste}$</td>
</tr>
<tr>
<td>L0N0</td>
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<td>1.50e-14</td>
</tr>
<tr>
<td>L1N0</td>
<td>1.49e-03</td>
<td>1.03e-03</td>
</tr>
<tr>
<td>L2N0</td>
<td>2.24e-03</td>
<td>1.46e-03</td>
</tr>
<tr>
<td>L3N0</td>
<td>6.52e-03</td>
<td>2.15e-03</td>
</tr>
<tr>
<td>L4N0</td>
<td>9.37e-03</td>
<td>3.14e-03</td>
</tr>
</tbody>
</table>

Table: Relative errors for PPE and STE methods.

\[ err_{ppe} = \frac{\|q_{ppe} - p_{ref}\|_{L^2}}{\|p_{ref}\|_{L^2}}, \quad err_{ste} = \frac{\|q_{ste} - p_{ref}\|_{L^2}}{\|p_{ref}\|_{L^2}} \]
Random error in the data

- two sources of error in velocity field:
  - interpolation error due to the limited amount of points with known velocity
  - velocity vectors are measured with the error/noise

- to simulate the latter one, we add a random number
  \( \varepsilon \in [-0.05, 0.05], \varepsilon \in [-0.1, 0.1] \) respectively, to each point where we know the velocity

- \( \mathbf{v}_{\text{exact}}(\mathbf{x}) \approx \mathbf{v}_{\text{meas}}(\mathbf{x}) = (1 \pm \varepsilon(\mathbf{x})) \mathbf{v}_{\text{exact}}(\mathbf{x}) \)
  - \( \varepsilon(\mathbf{x}) \in [0, 0.05] \) for maximal 5% error
  - \( \varepsilon(\mathbf{x}) \in [0, 0.1] \) for maximal 10% error
Random error in the data
Random error in the data

Convergence curves of relative errors for coarse data with 5% noise.

Convergence curves of relative errors for coarse data with 10% noise.
Comparison of two methods

PPE
- poisson equation
- additional derivative of the data vector $f(v)$
- underestimate the pressure difference
- fixing $p$ at the outlet
- limiting accuracy with noisy velocity fields

STE
- larger linear problem
- less regularity requirements on the data (velocity field)
- better approximation $\|q_{ste} - p\|_{L^2}$
- fixing $p$ at the outlet
- limiting accuracy with noisy velocity fields
Motivation

Current medical methods in stenosis evaluation and pressure determination

Pressure determination - continuum mechanics approach

Pressure difference and energy dissipation dependence on the stenosis severity
Energy dissipation

The parts of the geometry (from below): left ventricular outflow tract, ventriculo-aortic junction, aortic root (the first third should be stenotic), sinotubular junction, ascending aorta.
Model

\[ \rho_\ast \left( \frac{\partial v}{\partial t} + (\nabla v) v \right) - \text{div} \left( 2\mu_\ast D(v) \right) + \nabla p = 0 \]

\[ \text{div} \ v = 0 \]

\[ T = -pI + 2\mu_\ast D \]

\[ v = 0 \]

\[ v = v_{in} \]

\[ Tn - \frac{1}{2} \rho_\ast (v \cdot n) - v = -\overline{P(t)}n \]

in \( \Omega \),

in \( \Omega \),

in \( \Omega \),

on \( \Gamma_{wall} \),

on \( \Gamma_{in} \),

on \( \Gamma_{out} \).
Boundary conditions

Inflow velocity magnitude $V(t)$ and outlet pressure $P(t)$ as functions of time.
**Velocity fields**

**NO STENOSIS**

**50% STENOSIS**

Energy dissipation
**Pressure and energy dissipation on severity**

\[ p = \frac{\int_{\Gamma_z} p \, dS}{\text{area}(\Gamma_z)} \]

\[ E_{dis} = \frac{\int_{\Gamma_z} \mathbf{T} : \mathbf{D} \, dS}{\text{area}(\Gamma_z)} \]
Pressure difference estimation

Table: Maximal pressure difference computed through the Navier-Stokes eq. and simplified Bernoulli eq.
Conclusions

- The current methods for stenosis evaluation (whether this is physiologically important or not) are based on pressure difference computed through the simplified Bernoulli equation (pressure difference proportional to $v^2$).

- Continuum mechanics models are available to determine the pressure from the velocity field.

- The presented method, leading to the Stokes equation, allows us to compute the pressure under lower regularity requirements on the given velocity and it was shown that it provides more accurate pressure approximation.

- The pressure and energy dissipation were computed in the geometries with narrowing up to 80% - knowing the velocity field at the inlet (the left ventricular outflow) and pressure at the outlet (the ascending aorta).
ONGOING PROJECTS - PRESSURE DETERMINATION IN PATIENT-SPECIFIC GEOMETRIES

![Graph showing relative error vs. 1/average node distance for PPE and STE methods.](image1)

![Graph showing relative error vs. 1/average node distance for PPE and STE methods.](image2)
On-going projects - wall boundary condition

**NOSLIP**

\[ v \cdot n = 0 \]
\[ v \cdot \tau = 0 \]

**NAVIER SLIP K=0.5**

\[ v \cdot n = 0 \]
\[ Tn \cdot \tau = -\frac{1}{K} v \cdot \tau \]

**PERFECT-SLIP**

\[ v \cdot n = 0 \]
\[ Tn \cdot \tau = 0 \]

**Figure:** Velocity distribution on a slice of the valvular geometry without severity in time of maximal velocity (\( t = 0.15 \text{ s} \)): all plug-in profiles.
Ongoing projects - wall boundary condition

NOSLIP 30% severity

SLIP 30% severity

Figure: Velocity distribution on a slice of the valvular geometry with 30% severity in time of maximal velocity ($t = 0.15\ s$).
Ongoing projects - wall boundary condition

NOSLIP 50% severity

SLIP 50% severity

Figure: Velocity distribution on a slice of the valvular geometry with 50% severity in time of maximal velocity ($t = 0.15$ s).
**Ongoing projects - wall boundary condition**

\[
\frac{1}{\text{SEP}} \int_{\text{SEP}} \int_{\Gamma} |\mathbf{v}| \, dS \, dt \\
\int_{\Gamma} dS
\]

velocity [m/s]

\[
\frac{1}{\text{SEP}} \int_{\text{SEP}} \int_{\Gamma} \rho^* \frac{\mathbf{v} \cdot \mathbf{v}}{2} \, dS \, dt \\
\int_{\Gamma} dS
\]

kinetic energy [Pa]

50% STENOSIS, time-averaged values (over SEP)
**ONGOING PROJECTS - WALL BOUNDARY CONDITION**

\[
\frac{1}{\text{SEP}} \int_{\text{SEP}} \int_{\Gamma} p \, dS \, dt \quad \frac{1}{\text{SEP}} \int_{\text{SEP}} \int_{\Gamma} 2\mu \ast \mathbf{D} : \mathbf{D} \, dS \, dt
\]

**Pressure [mmHg]**

<table>
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<th>z coordinate [mm]</th>
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<tbody>
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<td>102</td>
</tr>
<tr>
<td>100</td>
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<tr>
<td>98</td>
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<tr>
<td>0</td>
</tr>
<tr>
<td>10</td>
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<tr>
<td>20</td>
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</tbody>
</table>

**Energy Dissipation [Pa/s]**

<table>
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<tbody>
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50% STENOSIS, time-averaged values (over SEP)
Thank you for your attention.