Stabilised finite element methods for ill-posed problems with conditional stability

Erik Burman
University College London

Stabilised finite element methods has emerged as an efficient and reliable tool for the design of computational methods for complex flow problems. Indeed even for well-posed partial differential equations a naive approach to the discretisation can lead to an ill-posed or ill-conditioned discrete problem and associated approximate solutions with poor accuracy. Adding stabilisation terms with suitable weak consistency properties can rectify the situation leading to optimally converging approximations. Many problems in fluid mechanics however are ill-posed on the continuous level, but nevertheless their numerical approximation is of great interest. This is the case for instance in weather-forcasting where initial or boundary data may be unknown. In this case typically so called Tikhonov regularisation is applied, often to the difference between the approximation and a background state that is assumed known. In this talk we will consider a particular class of ill-posed problems for which a so called conditional stability holds. This class includes model data assimilation problems and Cauchy type inverse problems for linear elliptic and parabolic problems. We will show how stabilised methods known from the literature may be applied for the approximation, or reconstruction, of these solutions, without the need of a background state. In the case of piecewise affine approximation we present error estimates that are optimal with respect to the approximation order of the space and the conditional stability estimate of the ill-posed problem. The effect of perturbations in data will also be discussed.