On the motion of a rigid body with a cavity filled with a viscous liquid

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We study inertial motions of the coupled system, S, constituted by a rigid body containing a cavity that is completely filled with a viscous liquid. We show that for data of arbitrary size (initial kinetic energy and total angular momentum) every weak solution (a la Leray-Hopf) converges, as time goes to infinity, to a uniform rotation, thus proving a famous conjecture of N.Ye. Zhukovskii. Moreover we show that, in an appropriate range of initial data, this rotation must occur along the central axis of inertia of S with the larger mo- ment of inertia. Furthermore, we provide necessary and sufficient conditions for the rigorous nonlinear stability of permanent rotations, which improve and/or generalize results previously given by other authors under different types of ap- proximation of the original equations, and/or suitable symmetry assumptions on the shape of the cavity. Finally, we present a number of results obtained by a targeted numerical simulation that, on the one hand, complement the analytical findings, whereas, on the other hand, point out new features that the analysis is yet not able to catch, and, as such, lay the foundation for interesting and challenging future investigation.