On Scheduling Problems with Forbidden Stack-Overflows

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Definition

n jobs $j = 1,2,\ldots,n$

m *stacks* $k = 1,2,\ldots,m$

p_j processing time of job j

d_j due date of job j

h_{jk} volume of sub-products, which is put to stack k after the completion of job j.

H_k^{max} capacity of stack k

A stack becomes less filled by 1 unit within a time unit.

Stack overflows are forbidden!

Scheduling problems to minimize makespan

$C_{max}(\pi)$ – *maximal completion time of the jobs in the schedule* π

$H2||C_{max}$ – *scheduling problem with m=2 stacks to minimize the makespan*

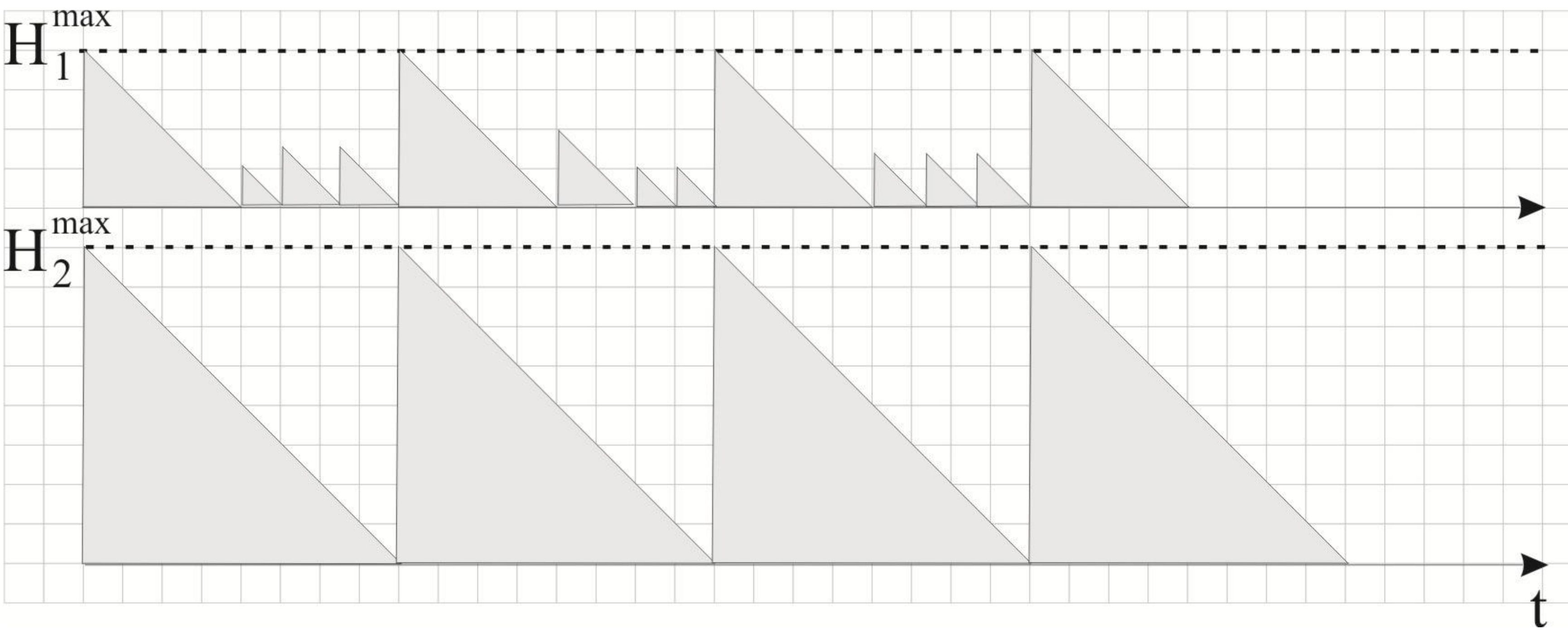
Lemma 1: Problem $H2||C_{max}$ **is NP-hard in the strong sense.**

Reduction from the 3-Partition problem:

$3\overline{m}$	*numbers* b_j *in the 3-Partition problem*
$3\overline{m} + \overline{m} + 1$	*jobs*
$h_{j1} = b_j$ and $h_{j2} = 0$	*for the first* $3\overline{m}$ *jobs*
$h_{j1} = B = H_1^{max}$ and $h_{j2} = 2B = H_2^{max}$	*for the next* $3\overline{m}+1$ *jobs*

Scheduling problems to minimize the makespan

Lemma 1: Problem *$H2||C_{max}$* is NP-hard in the strong sense.



$C_j = (j\text{-}n\text{-}1)2B$, $j = n+1, n+2, \ldots, n+\overline{m}+1$

Jobs from N_j are processed in the interval $[C_j, C_j + 2B]$, $j = n+1, n+2, \ldots, n+\overline{m}+1$

$p_j = 0$

Scheduling problems to minimize the makespan

Lemma 2: Problem $H2|h_{jk}=1|C_{max}$ is NP-hard.

Reduction from the Graph coloring problem:

For each vertex $v \in V$, we define a job j_v.

For each arc $(v,u) \in E$, we define a stack $k_{v,u}$

$Hk_{v,u}^{max} = 1$

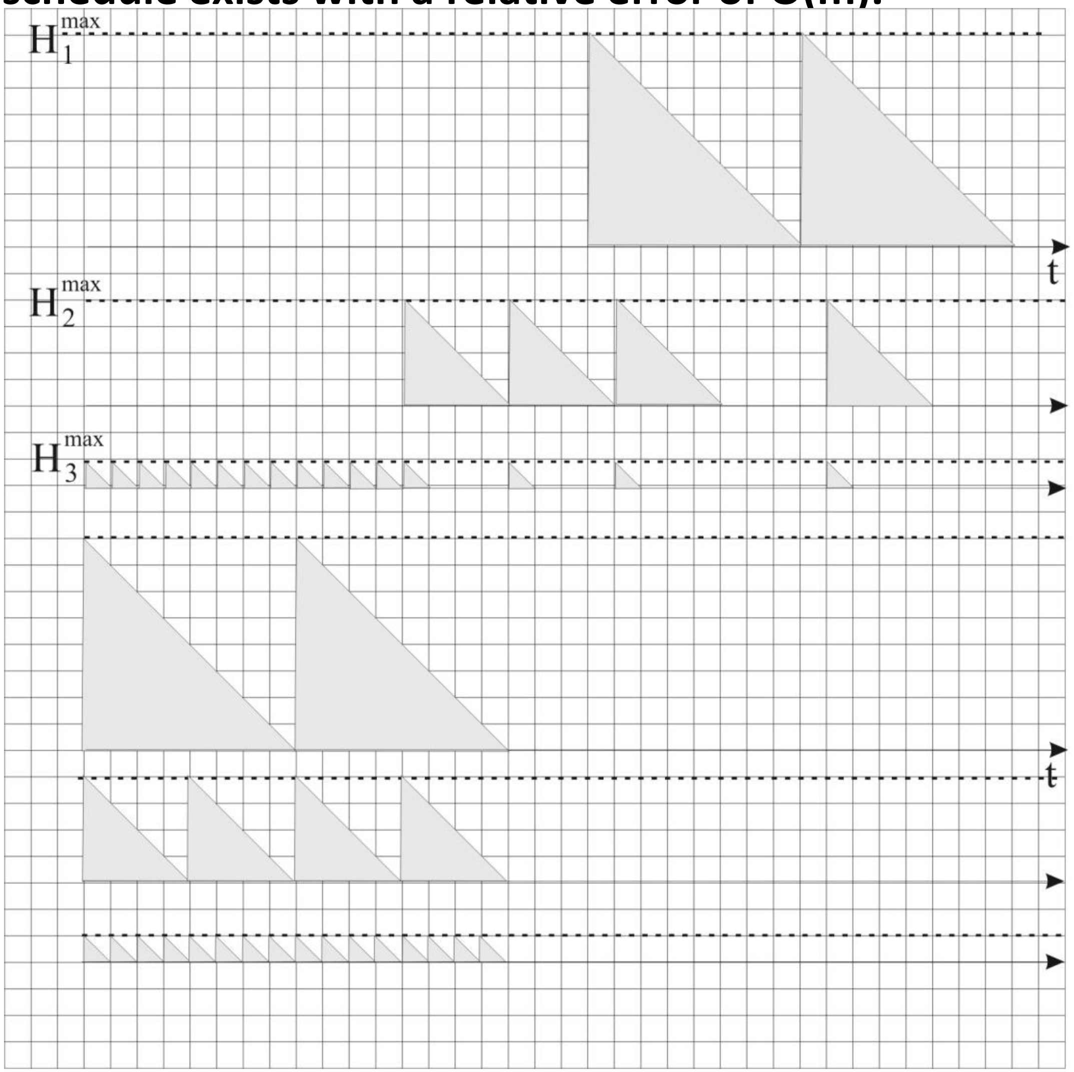
$$h_{j_v k_{v,u}} = h_{j_u k_{u,v}} = 1$$

Problem $Hm|h_{jk}=1|C_{max}$ is equivalent to:

- a special case of the Resource-Constrained Project Scheduling Problem with equal-length jobs and resource capacities equal to 1 without precedence relations;
- a special case of the School Timetabling Problem.

Scheduling problems to minimize the makespan

Lemma 3: There exists an instance of the problem *Hm*||C_{max} for which an active schedule exists with a relative error of O(m).



$$N = N_1 \cup N_2 \cup \ldots \cup N_m$$

Jobs from N_i heat only machines i, i+1, …, m

$$A \in Z^+$$

$$|N_i| = A^i - A^{i-1}$$

$$H_i^{max} = A^{m-i}$$

$$h_{jk} = H_k^{max}, \mathrm{k} \geq i$$

$$C_{max}(\pi) = A^m - 1$$
$$C_{max}(\pi') = m(A^m - A^{m-1})$$

Relative error:

$$m - 1 + \frac{m - mA^{m-1}}{A^m - 1}$$

Scheduling problems to minimize total tardiness

$T_j(\pi) = max\{0, C_j(\pi) - d_j\}$ – *tardiness of job j in the schedule π.*

$H1||\sum T_j$ - scheduling problem with m=1 stack to minimize total tardiness.

Lemma 4: Problem $H1||\sum T_j$ is NP-hard.

Reduction from the NP-hard special case of problem $1||\sum T_j$.

Scheduling problems to minimize total tardiness

$$\begin{cases} p_1 > p_2 > \cdots > p_{2n+1}, \\ d_1 < d_2 < \cdots < d_{2n+1}, \\ d_{2n+1} - d_1 < p_{2n+1}, \\ p_{2n+1} = M = n^3 b, \\ p_{2n} = p_{2n+1} + b = a_{2n}, \\ p_{2i} = p_{2i+2} + b = a_{2i}, \ i = n-1, \ldots, 1, \\ p_{2i-1} = p_{2i} + \delta_i = a_{2i-1}, \ i = n, \ldots, 1, \\ d_{2n+1} = \sum_{i:=1}^{n} p_{2i} + p_{2n+1} + \frac{1}{2}\delta, \\ d_{2n} = d_{2n+1} - \delta, \\ d_{2i} = d_{2i+2} - (n-i)b + \delta, \ i = n-1, \ldots, 1, \\ d_{2i-1} = d_{2i} - (n-i)\delta_i - \varepsilon\delta_i, \ i = n, \ldots, 1, \end{cases}$$

where $\delta_i \in Z^+, i = 1, 2, \ldots, n$, are integer numbers,

$$\delta = \sum_{i=1}^{n} \delta_i, \qquad b = n^2 \delta$$

and

$$0 < \varepsilon < \frac{\min_i \delta_i}{\max_i \delta_i}.$$

we add two jobs 2n+2 and 2n+3, where

$p_{2n+2} = p_{2n+1}$, $d_{2n+2} = 0$,

$p_{2n+3} = p_1 - p_{2n+1} < p_{2n+1}$ $d_{2n+3} = 0$

Thanks for your attention

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